

This is a closed book closed notes exam.

Attempt all problems. Write solutions on these sheets. Ask for scratch paper if the fronts and backs of these pages are not sufficient; put your name on any such extra sheets and hand them in with your exam.

Credit for an answer may be reduced if a large amount of irrelevant or incoherent material is included along with the correct answer.

Questions begin on the next sheet. Fill in your name on this sheet now, but do not turn the page until the signal is given. At the end of the exam, stop writing and close your exam as soon as the ending signal is given, or you will lose points.

Think clearly, stay calm.

Your name _____

Leave blank for grading

1, 2(a)	/ 30
2(b-d)	/ 30
3(a)	/ 20
3(b)	/ 20
Σ	/100

FAMILY CIRCUS *Bil Keane*



"I can't pull it off, Daddy. Is this one of those child-proof caps?"

1. (20 points, 10 points each.) Complete the following definitions. You may use, without defining them, any terms or symbols that our text defines before defining the word or symbol asked for. Your definitions do not have to have exactly the same wording as those in the text, but for full credit they should be clear, and mean the same thing as those definitions. More space is provided for the answers than a concise answer is likely to need.

(a) If Λ is an index set, and if for each $\alpha \in \Lambda$ we are given a set X_α , then $\bigcap_{\alpha \in \Lambda} X_\alpha$, the intersection of the X_α , is the set of those x which

(b) The *maximal principle* states that if \mathcal{F} is _____,

and if _____

then _____

2. (40 points, 10 points each.) For each of the items listed below, either *give an example* with the property stated, or give a brief reason why *no such example exists*.

If you give an example, you do *not* have to prove that it has the property stated; however, your examples should be specific; i.e., even if there are many objects of a given sort, you should name a particular one. If you give a reason why no example exists, don't worry about giving reasons for your reasons; a simple statement will suffice.

(a) An expression for 1 as a linear combination of x^2 , $(x+1)^2$ and $(x+2)^2$ in $P_2(R)$ (the vector space over R of polynomials of degree ≤ 2).

(2, continued)

(b) Five distinct elements v_1, v_2, v_3, v_4, v_5 in \mathbb{R}^2 such that $\text{span}(\{v_1, v_2\}) = \text{span}(\{v_3, v_4, v_5\})$.

(c) A 3-element set that spans $\mathbb{P}_3(\mathbb{R})$.

(d) A linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}$ such that $N(T) = \text{span}(\{(1,1,1), (0,1,2)\})$.

3. (40 points, 20 points each) Prove the following statements. You may assume all results proved in our text in the readings so far.

(a) If W_1 and W_2 are subspaces of a vector space V , then $W_1 + W_2$ (defined as $\{x + y : x \in W_1, y \in W_2\}$) is also a subspace of V .

(This is a result I said you could take for granted in one of the homework exercises. Now I am asking you to supply the details.)

(3, continued)

(b) Suppose V_1 , V_2 , V_3 are finite-dimensional vector spaces over a field F , and

$$T_1: V_1 \rightarrow V_2, \quad T_2: V_2 \rightarrow V_3$$

are linear transformations, such that T_1 is one-to-one, T_2 is onto, and $R(T_1) = N(T_2)$. Then

$$\dim(V_1) - \dim(V_2) + \dim(V_3) = 0.$$