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155 Dwinelle Hall

Fall 2008, Math 1B
First Midterm

19 Sept., 2008
3:10-4:00 PM

This is a closed book exam. You are allowed one 2-sided 8½" × 11" sheet of notes.

Attempt all problems. Write solutions on these sheets. Ask for scratch paper if the fronts and backs of these pages are not sufficient; put your name on any such extra sheets and hand them in with your exam.

Credit for an answer may be reduced if a large amount of irrelevant or incoherent material is included along with the correct answer.

Questions begin on the next sheet. Fill in your name and section on this sheet now, but do not turn the page until the signal is given. At the end of the exam, stop writing and close your exam as soon as the ending signal is given, or you will lose points.

Think clearly, stay calm.

Your name _____

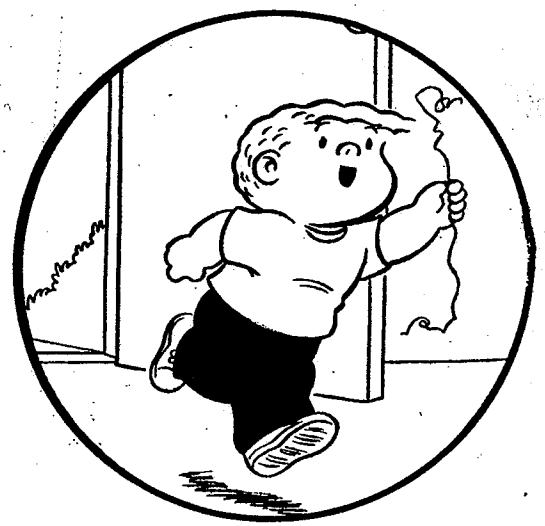
Sections: Mark yours with ×.
(Note that they are listed in order of hour, not section-number.)

usual place, hour (MW),		Sec.		TA
171 Stanley	8:00 - 9:00	**201**	☐	Benjamin Tsou
3102 Etcheverry	9:00-10:00	**203**	☐	Kiril Datchev
71 Evans	10:00-11:00	**204**	☐	Benjamin Tsou
3111 Etcheverry	11:00-12:00	**205**	☐	Harold Williams
75 Evans	12:00 - 1:00	**206**	☐	Koushik Pal
70 Evans	1:00 - 2:00	**207**	☐	Gary Sivek
105 Latimer	2:00 - 3:00	**208**	☐	Gary Sivek
3102 Etcheverry	2:00 - 3:00	**211**	☐	Koushik Pal
85 Evans	5:00 - 6:00	**210**	☐	Harold Williams
	Other or none		☐	Explain _____

Leave blank for grading

1(a-c)	/ 36
1(d,e)	/ 29
2	/ 20
3	/ 15
Σ	/100

FAMILY CIRCUS *Bil Keane*



"Mr. Pizzarelli gave me his old guitar string! Can I learn how to play it?"

1. (65 points: 12 points for each except (d), which is 17 points.) Compute the following integrals.

work | *answers:*

(a) $\int x e^{-2x} dx$

(a)

(b) $\int \sin^5 x \cos^6 x \, dx$

(b)

(c) $\int_{-\pi/4}^{\pi/4} \tan^2 x \, dx$

(c)

(1, continued)

work *answers:*

(d) $\int \frac{(x+1)^2}{x^2 - 3x}\, dx$

(d)

(e) $\int_{-\infty}^{-1/1000} x^{-5/3}\, dx$

(e)

2. (20 points) One of the formulas in the table of integrals in the back of our book is:

$$\textbf{25.}\ \int \frac{du}{\sqrt{a^2+u^2}} = \ln(u + \sqrt{a^2+u^2}) + C.$$

Use the above formula to obtain a formula for $\int \frac{dx}{\sqrt{x^2+px+q}}$, where p and q are real constants. (Hint: complete the square.) Indicate what inequality p and q must satisfy for this formula to follow from the formula given. Fill in your final answers as indicated at the bottom of this page.

Answer: $\int \frac{dx}{\sqrt{x^2+px+q}} =$ _____

Inequality that must be assumed _____

3. (15 points) Our text states the Midpoint Rule:

$$\int_a^b f(x)\,dx \approx \Delta x[f(\bar{x}_1) + \ldots + f(\bar{x}_n)], \text{ where } \Delta x = (b-a)/n \text{ and } \bar{x}_i = (x_{i-1} + x_i)/2.$$

For this rule, it gives the error estimate saying that if $|f''(x)| \le K$ for $a \le x \le b$, then

$$|E_M| \le K(b-a)^3/24n^2.$$

Compute the bound that this gives for the approximation

$$\int_0^2 \sin 5x\,dx \approx (2/100)\,(\sin(5\cdot 0.01) + \sin(5\cdot 0.03) + \sin(5\cdot 0.05) + \ldots + \sin(5\cdot 1.99)).$$

Namely, if the above approximation to the integral is denoted M, determine numerically what range of values around M the above error estimate says that $\int_0^2 \sin 5x\,dx$ must lie in. Use the exact value coming from the error estimate, not a decimal approximation. Fill in your final answer as indicated at the bottom of this page.

Answer: _____ $\le \int_0^2 \sin 5x\,dx \le$ _____.