George M. Bergman | Fall 2008, Math 1B | 16 Dec., 2008
155 Dwinelle Hall | **Final Examination** | 12:30-3:30 PM

This is a closed book exam. You are allowed one 2-sided 8½" × 11" sheet of notes.

Attempt all problems. Write solutions on these sheets. Ask for scratch paper if the fronts and backs of these pages are not sufficient; put your name on any such extra sheets and hand them in with your exam.

Credit for an answer may be reduced if a large amount of irrelevant or incoherent material is included along with the correct answer.

Questions begin on the next sheet. Fill in your name and section on this sheet now, but do not turn the page until the signal is given. At the end of the exam, stop writing and close your exam as soon as the ending signal is given, or you will lose points.

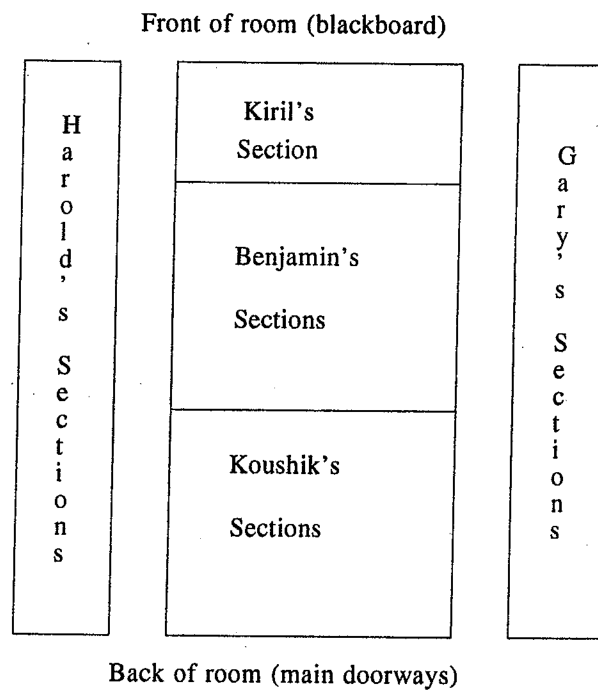
Think clearly, stay calm.

Your name ______________________________

Sections: Mark yours with ×.
(in order of hour, not section-number.)

usual place, hour (MW),		Sec.		TA
171 Stanley	8:00 - 9:00	**201**	☐	Benjamin Tsou
3102 Etcheverry	9:00 - 10:00	**203**	☐	Kiril Datchev
71 Evans	10:00-11:00	**204**	☐	Benjamin Tsou
3111 Etcheverry	11:00-12:00	**205**	☐	Harold Williams
75 Evans	12:00 - 1:00	**206**	☐	Koushik Pal
70 Evans	1:00 - 2:00	**207**	☐	Gary Sivek
105 Latimer	2:00 - 3:00	**208**	☐	Gary Sivek
3102 Etcheverry	2:00 - 3:00	**211**	☐	Koushik Pal
85 Evans	5:00 - 6:00	**210**	☐	Harold Williams
Other or none			☐	Explain ____________

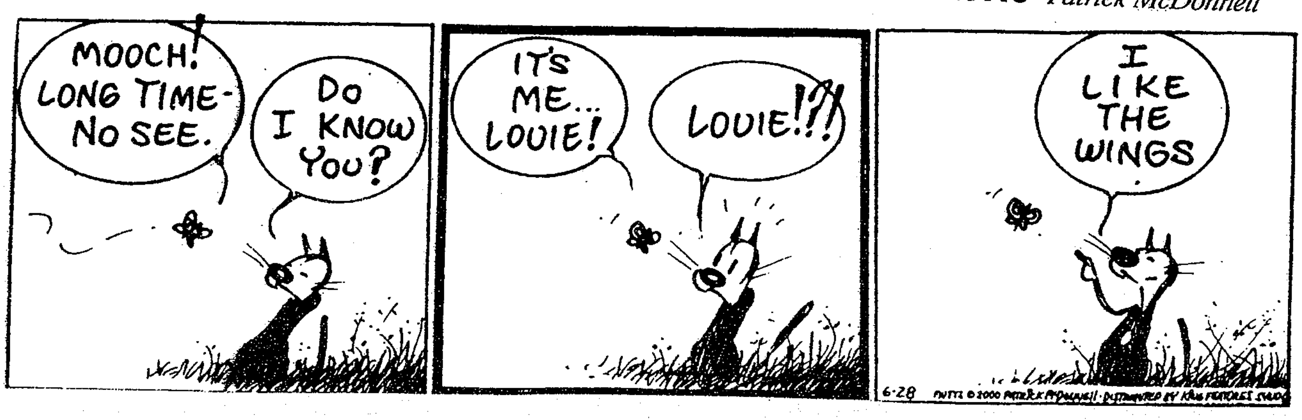
SEATING PLAN



Leave blank for grading

1(a-c)	/ 12
1(d-f)	/ 12
2(a,b)	/ 14
2(c,d)	/ 14
3	/ 21
4	/ 14
5	/ 13
Σ	/100

MUTTS *Patrick McDonnell*



1. (24 points: 4 points each.) Compute the following.

work | *answers:*

(a) $\int dx/(x^3+x)$.

(a)

(b) $\Sigma_{n=0}^{\infty} (2^n + 3^n)/n!$

(b)

(c) The set of real numbers x such that $\Sigma_{n=0}^{\infty} n^5 x^n$ converges.

(c)

(**1**, continued)

work | *answers:*

(d) The general solution to the differential equation $y' = x^2y^3$.

(d)

(e) The general solution to the differential equation $y'' + 4y' - 21y = 0$.

(e)

(f) The general solution to the differential equation $2y'' + y = x^2$.

(f)

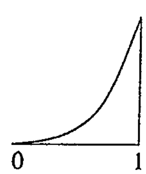
2. (28 points: 7 points each.) Compute the following.

work | *answers:*

(a) An arc-length function for the curve $y = e^x$ (i.e., a function $F(x)$ such that for $a < b$, the length of the curve from $x = a$ to $x = b$ is given by $F(b) - F(a)$).

(a)

(b) The centroid of the region bounded above by the curve $y = x^n$, below by the x-axis, and on the right by the line $x = 1$; where n is a positive integer.



(b)

(**2**, continued)

work | *answers:*

(c) The solution to the differential equation $y' = y \sin x + \sin x$ satisfying $y(0) = 0$.

(c)

(d) The general solution to the differential equation $y'' - 2y' - 3y = e^{3x} + 10 \sin x$.

(d)

4. (14 points) Prove that if $a_0, a_1, a_2, \ldots, a_n, \ldots$ is a bounded sequence of real numbers, then the series $\Sigma_{n=0}^{\infty} a_n/2^n$ is absolutely convergent.

In proving this, you may call on any facts about series in our text.

5. (13 points) Recall that if f is a function with $n+1$ derivatives on an interval I, and a is a point of I, then the n-th degree Taylor approximation of $f(x)$ about a means the polynomial

$$T_n(x) = \Sigma_{i=0}^{n} f^{(i)}(a)\,(x-a)^i/i!,$$

and Taylor's formula says that for every point x of I, the remainder $R_n(x) = f(x) - T_n(x)$ equals $f^{(n+1)}(z)\,(x-a)^{n+1}/(n+1)!$ for some z between a and x.

(I have slightly simplified the statement in the text, removing the assumption that x is distinct from a, and hence the statement that z lies *strictly* between a and x. It remains true in this simplified form.)

Write out the second degree Taylor approximation for e^x about $a = 2$, and verify that for all $x \in [1.5, 2.5]$, Taylor's formula shows that this polynomial approximates e^x to within $\leq e^{5/2}/48$.

$T_2(x)$ = ____ .

Verification of error bound: