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Fall 2005

Math 250a Midterm

50 minutes

1. (25 points) Define the following. You do not need to define terms used in those definitions.

   (a). Monoid.

   (b). Group action.

   (c). Inverse (projective) limit $\varprojlim$ in a category. Do not refer to the corresponding definition for groups.

2. (25 points) Determine all subgroups of $\mathbb{Z}$.
   (This was proved in the book. In proving it here, you may use any results proved in the book, up to the point where that result was obtained.) (You may *not* assume that all subgroups of $\mathbb{Z}$ are cyclic.)

3. (25 points) In a *nonabelian* group, there may be elements x and y of finite order, whose product xy has infinite order. Therefore the set of elements of finite order may not be a subgroup (you do not need to prove this or give an example).
   One can, however define a **torsion subgroup** to be a subgroup, all of whose elements have finite order.
   Show that a (possibly infinite) group G has a maximal torsion subgroup (i.e., a torsion subgroup that is maximal among all torsion subgroups, not necessarily among all subgroups of G). Here "maximal" refers to the inclusion ordering.

4. (25 points) Let
   $$G = (\mathbb{Z}/m\mathbb{Z}) \oplus (\mathbb{Z}/n\mathbb{Z})$$
   with $n \mid m$, and let $H < G$ be the subgroup corresponding to the first factor above (i.e., $H = (\mathbb{Z}/m\mathbb{Z}) \oplus 0$, so it's *all of* the first factor). Determine (with proof) how many subgroups K of G have the property that $G = H \times K$ (as in Prop. 2.1 in the book; i.e., *internal* product).