MATH H110

PROFESSOR KENNETH A. RIBET

Second Midterm Exam

November 3, 2003

12:10–1:00 PM

Name:

SID:

Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. You may refer to a single 2-sided sheet of notes. Please write your name on each sheet of paper that you turn in; don't trust staples to keep your papers together. Explain your answers in full English sentences as is customary and appropriate. Your paper is your ambassador when it is graded.

Problem:	Your score:	Total points
1		7 points
2		9 points
3		14 points
Total:		30 points

1. Let V be a vector space over a field F and let v be a vector in V. Let $T\colon V \to V$ be a linear transformation. Suppose that $T^m(v) = 0$ for some positive integer m but that $T^{m-1}(v)$ is non-zero. Show that the span of $\{ v, T(v), T^2(v), \ldots, T^{m-1}(v) \}$ has dimension m.

2. Let $T\colon V \to V$ be a linear map on a non-zero finite-dimensional vector space V over a field F. Suppose that the characteristic polynomial of T splits over F into a product of linear factors. Show that there is a basis B of V such that $[T]_{\mathsf{B}}$ is upper-triangular.

3. Let V be an n-dimensional real or complex inner product space. Let $e_1, \ldots, e_n$ be an orthonormal basis of V. Suppose that $T\colon V \to V$ is a linear transformation and let $T^*\colon V \to V$ be the adjoint of T. Show that $\sum_{j=1}^{n} \|T^*(e_j)\|^2 = \sum_{j=1}^{n} \|T(e_j)\|^2$.

(continuation)
If $\|T^*v\| \leq \|Tv\|$ for all $v \in V$, show that $\|T^*v\| = \|Tv\|$ for all $v \in V$.