

D. Geba MIDTERM - MAT 104 (10/10/03)

① Let the sequence  $(x_n)$  be defined by:  
 $x_1 = 1$ ,  $x_{n+1} = 1 + x_n^2$ . Prove that  $(x_n)$  is monotonous and find its limit.

② Let the sequence  $(x_n)$  be defined by:  
 $x_n = \frac{a^n}{a^n + b^n}$  where  $a, b > 0$  are fixed numbers  
 Prove that  $(x_n)$  is convergent and find its limit.

③ Let the sequence  $(a_n)$  be defined by:  

$$a_n = \frac{1}{1+2+\dots+n}$$

a) Prove by induction that  $a_n = \frac{2}{n(n+1)}$ ,  $\forall n \geq 1$ .

b) Prove that the series  $\sum_{n=1}^{\infty} a_n$  is convergent.

c) Find the sum of the series at b).

④ Let  $f: \mathbb{R} \rightarrow \mathbb{Z}$  be a continuous function.  
 Prove that  $f$  is constant i.e.  $\exists c \in \mathbb{Z}$  such that  $f(x) = c$ ,  $\forall x \in \mathbb{R}$ .