

MATH 410-6

Midterm I

- Let $(V, +, \cdot)$ be a vector space, $\{u_1, \dots, u_m\} \subset V$ and $\{v_1, \dots, v_m\} \subset V$. Prove:
 $\{u_1, \dots, u_m\}$ spans V , $\{v_1, \dots, v_m\}$ linearly independent $\Rightarrow m \leq \dim V$
- Prove that if V and W are three-dimensional subspaces of \mathbb{R}^5 , then V and W must have a nonzero vector in common.
- Find a basis for the vector space of 3×3 symmetric matrices.
- Let $(V, +, \cdot)$ be a vector space, $\{v_1, v_2, v_3\} \subset V$ linearly independent. Analyse the linear independence/dependence of $\{w_1, w_2, w_3\}$ for $w_1 \equiv v_1 + v_2$, $w_2 \equiv v_1 + v_3$, $w_3 \equiv v_2 - v_3$