

Math 115 Final 2003 Dec 17 12:30-3:30. Prof Borchers

Please make sure that your name is on everything you hand in.

You are allowed calculators and 1 page of notes.

All questions have about the same number of marks.

1. Find integers x and y such that $71x - 50y = 1$.
2. Find all pairs of positive integers a and b with greatest common divisor 6 and least common multiple 144.
3. What are the possible units digits (in base 10) of fourth powers?
4. Find the number of positive integers less than 3000000 that are coprime to 3000000.
5. Find all primitive roots of 23.
6. For each of the following, determine whether the form is positive definite, negative definite, or indefinite:
 - (a) $x^2 - 3xy + y^2$
 - (b) $-3x^2 + 4xy - 2y^2$.
7. Evaluate the Jacobi symbol $\left(\frac{976}{97}\right)$.
8. Prove that if the integer n is a sum of 3 squares, then n is not congruent to 7 mod 8. Find two integers m and n that are both sums of 3 squares whose product is not a sum of 3 squares.
9. Find the three positive definite reduced binary quadratic forms $ax^2 + bxy + cy^2$ of discriminant -31 .
10. Find integers x and y such that $x^2 + y^2 = 1009$. (The number 1009 is a prime.)