

**MATH 160 FINAL**  
**DECEMBER 18, 2003**  
H. Wu

**Your Name:** \_\_\_\_\_

I (48%; 8% each) For each of the names or formulas or concepts, explain its significance in the history of mathematics and supply as much information as you can. Give dates whenever possible. Write <sup>on</sup> the back of the page if necessary.

(1) Elliptic planetary orbits.

(2) G. Cardano.

(3) J. Napier.

(4) G. Desargues.

(5)  $\sum_{i=1}^n i^k$  for a fixed positive integer  $k$ .

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(6)  $\sum_{n=1}^{\infty} \frac{1}{n^k}$  for a fixed integer  $k > 1$ .

II (15%) According to the cubic formula, the equation  $x^3 - 63x = 162$  has the following root:

$$\sqrt[3]{81 + 30\sqrt{-3}} + \sqrt[3]{81 - 30\sqrt{-3}}$$

This root should be an integer. Do you know what this integer is, and why?

III (15%) Let  $p$  be a prime integer. <sup>72</sup> (1) If  $p$  is a sum of two integer squares, prove that  $p \equiv 1 \pmod{4}$ . (2) Suppose  $a, b, c, d$  are positive integers and  $p = a^2 + b^2 = c^2 + d^2$ . Prove that  $\{a, b\} = \{c, d\}$ .

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IV (15%) Without getting the exact answer, explain how one can integrate  $\int \frac{x^2 dx}{(x^2 + 6x + 10)^2}$  in terms of elementary functions. Note:  $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$  and  $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$ .

V (7%) Write down what you consider to be notable things (if any) that you have learned in this course.