

Math 130 Midterm IF 2002 September 23, 2002 Wisc

1. (20%) Let T be the translation of \mathbb{R}^2 by $P = (3, -2)$ ($T(Q) = Q + P$ for every $Q \in \mathbb{R}^2$), let ρ be the rotation of $+90^\circ$ around the origin, and let R be the reflection with respect to the line $x - y = 0$. What is the point $(R \circ T \circ \rho)(1, 2)$?
2. (20%) Let R be the reflection with respect to a line L . Suppose L is a line so that $R(L) \parallel L$. Prove that $L \parallel L$.
3. (30%) Let L be a line in \mathbb{R}^2 and let $\mathbb{R}^2 = \mathbb{R}_+^2 \cup L \cup \mathbb{R}_-^2$ be the usual decomposition of \mathbb{R}^2 as a union of L and the two half-spaces of L . Suppose \mathcal{C}, \mathcal{A} are convex sets in \mathbb{R}^2 so that (1) $\mathcal{C}, \mathcal{A}, L$ are disjoint and (2) $\mathbb{R}^2 = \mathcal{C} \cup L \cup \mathcal{A}$. Prove that $\mathcal{C} = \mathbb{R}_+^2$ or \mathbb{R}_-^2 .
4. (25%) Let $-\pi \leq \theta \leq \pi$, and let ρ_θ with rotation of (oriented) angle θ around the origin. Prove that $\rho_\theta \circ \rho_{-\theta} = \text{id}$, where $\text{id} = \text{identity map of } \mathbb{R}^2$.
5. (5%) How many hours on average do you spend on this course per week? Of these, how many are devoted to doing the homework?