MAT 110 - MIDTERM 9/27/02

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1. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ the linear transformation defined by

$$T(x, y, z) = (x + y + z, x + 3y + 5z).$$

a) Find $N(T), R(T)$ and compute dim $N(T)$, dim $R(T)$.

b) Let β and γ the standard bases for $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. Consider also $\alpha = \{(1, 1, 1), (2, 3, 4), (3, 4, 6)\}$ basis for $\mathbb{R}^3$.

Compute Q the change of coordinate matrix from β to α and the representation matrices $[T]_\alpha^\gamma$,$[T]_\beta^\gamma$.

Check that

$$[T]_\alpha^\gamma \cdot Q = [T]_\beta^\gamma.$$

2. Let V and W two vector spaces over the rational numbers field $\mathbb{Q}$ and $T : V \to W$ which satisfies

$$T(x + y) = T(x) + T(y).$$

Prove that T is a linear transformation.

3. Let m and n two positive integers. Construct a linear transformation T such that nullity$(T) = m$ and rank$(T) = n$.

4. Let $T : V \to V$ a linear transformation, where V is a finite-dimensional vector space. Prove that if rank(T)=rank(T^2) then $R(T) \cap N(T) = \{0\}$.

5. Let A, B two square matrices, $A, B \in M_{n\times n}(\mathbb{R})$ such that $I_n - AB$ is invertible. Prove that $I_n - BA$ is invertible.