Math 16A, Summer 2009

Name:

Each Problem is worth 10 points.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	Total
Score													/ 120

Problem 1. For each question below, circle True or False. **DON'T GUESS**, 2 points for each right answer, minus 2 points for each wrong answer.

- (a) **True** or **False**: Suppose that two cars are racing. After t seconds the first car is traveling at velocity $v_1(t)$ feet per second and the second car at $v_2(t)$ feet per second. If the average value of $v_1(t)$ on [0, 60] is equal the average value of $v_2(t)$ on [0, 60], then the cars have travelled the same distance after 60 seconds.
- (b) True or False:

$$\int xe^x \, dx = e^x(x-1) + C.$$

(c) **True** or **False**: $y = 1 + e^{2x}$ satisfies the differential equation

$$y' = 2y.$$

(d) **True** or **False**:

$$\frac{d}{dx} \left[\int_{\frac{1}{2}}^{x} \ln t \, dt \right] \bigg|_{x=1} = 0.$$

(e) **True** or **False**: As n gets very large,

$$\left[\frac{1}{n} + \frac{2}{n} + \dots + \frac{n-1}{n}\right] \cdot \frac{1}{n}$$

approaches 1. (Hint: This is a Riemann sum.)

Problem 2. Compute the following indefinite integrals:

(a)

$$\int \sqrt{2x-3} \, dx$$

(b)

 $\int e^{4+3x} \, dx$

Problem 3. Compute the following definite integrals:

(a)

$$\int_0^1 \frac{3}{(4-2x)^2} \, dx$$

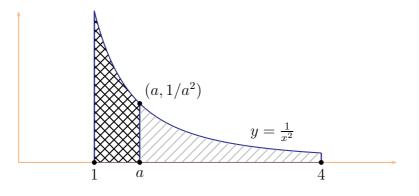
(b)

 $\int_{0}^{2} \frac{e^{x} - e^{-2x}}{2e^{x}} \, dx$

Problem 4. Suppose that the marginal revenue function for a company is 200 - 2x. Find the additional revenue received from doubling production if currently 10 units are being produced.

Problem 5. Compute the area of the region between the curves $y = x^2 - 1$ and y = x + 1 from x = 0 to x = 4.

Problem 6. For what value of a do the two shaded regions below have the same area?



Problem 7. Suppose that a lake is stocked with 100 fish. After 1 month, there are 150 fish in the lake. An ecological study predicts that the lake can support 600 fish. Use a logistic growth curve to estimate the number of fish in the lake after 1 year.

Problem 8.

(a) Find the percentage rate of change of the function $f(x) = 2x^2 - 3x$ at x = 2.

(b) Find the relative rate of change of the function $f(x) = \sqrt[3]{\frac{(2x+1)^2}{(2-x)e^x}}$ at x = 1.

Problem 9. A company can sell $q = \frac{2000}{p} - 200$ units of a particular commodity at a price of p dollars per unit.

(a) Compute the elasticity E(p).

(b) If the price is currently \$5 per unit, would the revenue increase or decrease when the price is raised?

Problem 10. Find the consumer's surplus for the demand curve $p = \frac{200}{x+10} + 10$ at sales level x = 10.

Problem 11. Use a Riemann sum with n = 4 and midpoints to estimate the area under the graph of y = 2x + 1 on the interval $2 \le x \le 4$.

Problem 12. Compute the volume of a frustrum of a right circular cone with height 1, lower base radius 2, and top radius 1, obtained by rotating the shaded region below about the *x*-axis:

