

Department of Mathematics, University of California, Berkeley

## Math 1A

Alan Weinstein, Fall 2000

**Second Midterm Exam, Tuesday, November 7, 2000  
(IT'S ELECTION DAY—DON'T FORGET TO VOTE)**

**Instructions.** Be sure to write on the front cover of your blue book: (1) your name, (2) your Student ID Number, (3) your TA's name (Eric Antokoletz, Victor Deletang, Matthieu Hamel, Andre Henriques, Di-An Jan, Chu-Wee Lim, Russell O'Connor, Alf Onshuus, Emmanuel Py, Shahed Sharif, Dan Stevens, Karla Westphal, or Alexander Woo).

Read the problems very carefully to be sure that you understand the statements. Show all your work as clearly as possible, and circle each final answer to each problem. **When doing a computation, don't put an "=" sign between things which are not equal. When giving explanations, write complete sentences.** If you have any extra time at the end of the exam, use it to check your work.

- 1 . [15 points] Sketch the graph

$$y = x/(1 + x^2),$$

labeling asymptotes, local extrema, and points of inflection. Mention any symmetries which the graph may have.

2 . [5 points] Let  $f(t)$  be a differentiable function which models the median price of an apartment in San Francisco at time  $t$ , measured in years (so that  $t$  is now approximately 2000.8). Consider the statement: "*The median price of apartments in San Francisco is expected to rise by less in the year 2001 than it did in 2000.*" For  $f$  to reflect this statement, what is the appropriate value of:

- (A) the sign (i.e. positive or negative) of  $f'(2001)$ ?  
(B) the sign of  $f''(2001)$ ?

You must give some reason for your answers.

3 . [5 points] State Highway 74 goes north-south, while Highway 73 goes east-west. They intersect at the town of Nader. At noon, George Bush leaves the town of Buchanan, which is 60 miles east (i.e. to the right) of Nader and drives west for 2 hours on Highway 73 at 74 miles per hour. At the same time, Al Gore leaves the town of Clinton, which is 60 miles south of Nader, and drives north for 3 hours on Highway 74 at 73 miles per hour. Since they are going at different speeds, they just miss colliding. At what rate is the distance between Bush and Gore changing at the moment when they come closest to one another?

**EXAM CONTINUES ON THE OTHER SIDE OF THE PAGE**

4. [10 points] Find the following limits.

(A)

$$\lim_{x \rightarrow 1} \frac{e^x - e}{x^2 - 1}$$

(B)

$$\lim_{x \rightarrow \pi} \frac{\tan x - x + \pi}{x^3}$$

(C)

$$\lim_{x \rightarrow \infty} (e^{2x} + 1)^{\frac{1}{x}}$$

5. [10 points] Let  $f$  be a function such that  $f(x)$ ,  $f'(x)$ , and  $f''(x)$  are defined for all  $x$  on the real line. Suppose, additionally, that  $f(0) > 0$ ,  $f'(0) = 0$ , and  $f''(x) > 1$  for all  $x$ .

(A) Find an equation for the critical point(s) of the function  $f(x)/x$ .

(B) Sketch a graph of a function with the properties of  $f$ , and give a geometric interpretation of the equation in part (A). Specifically, determine the  $y$ -intercept of the tangent line to the graph of  $f$  at each point  $(x_0, f(x_0))$  for which  $x_0$  is a critical point of  $f(x)/x$ .

(C) How many critical points does  $f(x)/x$  have? A figure will suffice to justify your answer to this part.

(D) [for 3 bonus points] What kind of extrema are the critical points of  $f(x)/x$ ; i.e. are they local maxima or local minima? (You must justify your answer.)

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