

Anatole Katok
Penn State

New Progress in Measure Rigidity of Abelian Group Actions and Littlewood Conjecture in Diophantine Approximation

ABSTRACT

We outline parts of the ongoing effort to understand the structure of invariant measures for algebraic (homogeneous and affine) actions of higher rank abelian groups Z^n and R^n for $n > 1$) with hyperbolic behavior. These efforts started from attempts to answer the question concerning common invariant measures for multiplication by p and q on the circle (assuming that p and q do not have a common power) raised in the Furstenberg's 1967 paper and motivated by some questions about uniform distributions. During 1990's important conceptual and technical developments have taken place but until very recently the results were mostly of internal interest for the subject. This changed beginning with what looked as a further technical advance in our 2002 paper with M. Einsiedler. Building on the insights from that paper and a very ingenious construction involving Hecke operators E. Lindenstrauss was able to achieve a breakthrough in the problem of arithmetic quantum chaos. Some of Lindenstrauss' innovations in turn helped to advance the study of invariant measures for the Weyl chamber flow on $SL(n, R)/SL(n, Z)$ to the point that led to a very substantial advance toward the Littlewood conjecture about multiplicative Diophantine approximation. Description of this last advance made in the joint paper with Einsiedler and Lindenstrauss will be the main topic of the talk.