Some remarks on the preparation of mathematics teachers in China* (Executive Summary)

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*"China" in this chapter will refer specifically to the People's Republic of China, i.e., not including Hong Kong or Taiwan.

I am grateful to Larry Francis for his critical input.

The preceding chapters in the Li-Huang volume contain important information about the education of mathematics teachers in China. From an American perspective, the following two features in the Chinese preparation of mathematics teachers stand out:

(A) A strong emphasis on content knowledge in pre-service PD.

(B) Making professional growth and lifelong education integral parts of the teaching culture.

The pre-service preparation of mathematics teachers in China puts great emphasis on the acquisition of content knowledge (as in (A)), but gives less attention to the acquisition of experience in student teaching or *pedagogical content knowledge* (**PCK**). The rationale for the emphasis on content knowledge is that it is impossible to develop pedagogical content knowledge without appropriate content knowledge. Fair enough, but one has to know what kind of "content knowledge" we are talking about here. For elementary *mathematics* teachers, a survey of 16 major teacher-training programs in China shows that, in general, the required mathematics courses include:

> Mathematical Analysis, Spatial Analytic Geometry, Projective Geometry, Non-Euclidean Geometry, Theory of Probability, The Structure of Algebra, a Brief History of Mathematics, Mathematical Modeling, Advanced Algebra, and Elementary Number Theory.

For secondary teachers, the required courses include:

> Ordinary Differential Equations, Classical Geometry, Complex Analysis, Probability and Statistics, Abstract Algebra I and II, Differential Geometry, Number Theory, Real Analysis, and Combinatorics and Graph Theory.

It is easy to see that there is a sizable gap between the mathematics Chinese preservice teachers are required to learn and the mathematics these teachers will actually teach in their classrooms. Some elementary teachers have voiced their misgivings about this gap ("Some of the courses are so boring, also difficult", "I don't know why we need to learn this", etc.) Some deans of normal universities have tried to narrow this gap, but the issue of how to integrate mathematics content and practice-based mathematical pedagogical knowledge remains an ongoing challenge in China. There is an additional concern about this approach to content knowledge; see below.

On the other hand, this content issue is offset by the system of in-service professional development in China, which has a long tradition of supporting *professional growth and lifelong education* for the acquisition of classroom management skills and PCK (see (B) above). Once Chinese teachers start their teaching careers, they are required to actively and constantly engage in a variety of professional development activities, including pairing up with an experienced teacher of the same grade level for one-on-one mentoring, participating in teaching research groups in which teachers study textbooks and develop lessons together, and giving public lessons for colleagues and administrators to observe. To Chinese mathematics teachers, doing lesson preparation, observation, and post-lesson reflection together is a way of life.

Their American counterparts may likely read about such a teaching culture with longing and envy.

Although many Chinese educators consider the advocacy of (A) at the expense of PCK to be a weakness of their system, the opposite has occurred in the U.S. where there has been a longstanding over-emphasis on pedagogy over content knowledge. In this climate, (A) should come as a welcome correction.

Although (B) has not been central to current discussions about mathematics education in the U.S., it should be. There is a teacher dropout crisis at the moment. In each of the years from 1988 to 2013, between 12.4 and 16.5 percent of public school teachers moved to a different school or left the teaching profession entirely. More than 42% of new teachers leave teaching within 5 years of entry. The reasons vary, but one of them is apparently teachers' strong desire for learning from each other, a desire which often goes unmet in the teaching profession. It was almost as if teachers were asked whether they wanted (B) in their professional lives and they overwhelmingly voted *yes*! What these statistics point to is an undeniable need for the American teaching culture (at least in the public sector) to embrace some version of (B).

However, we cannot recommend (A) and (B) for the U.S. without some important qualifications (as of 2025) for at least two reasons. We will deal with the content issue first. In the U.S., the mathematical knowledge that our teachers bring to their classrooms has typically not been *mathematics* but a corrupted version of mathematics that has resided in the standard school textbooks for (at least) the past four decades. This body of knowledge is as different from *mathematics* as margarine is from butter. We call it **Textbook School Mathematics (TSM)**. The mathematics enshrined in TSM is unlearnable because of its lack of what may be called logical *transparency* and *accountability*. In greater detail, the unlearnability of TSM is the result of its pervasive violation of the following five **Fundamental Principles of Mathematics (FPM)**:

1. Every concept has a precise *definition*. Definitions leave no doubt about what exactly students have to learn.

2. Mathematical statements are *precise*. Precision eliminates the need to *guess* in learning mathematics.

3. Every assertion is supported by logical *reasoning*. Mathematics is learnable because it is reasonable.

4. Mathematics is *coherent*. Mathematics is a living organism rather than a bag of isolated tricks for students to memorize.

5. Mathematics is *purposeful*. The purpose lets students know where they are

headed.

The mathematics education reform of 1989 failed to produce non-TSM school textbooks, only textbooks that were on a different pedagogical platform. To a lesser extent, the same is true of the 2010 reform.

We will illustrate how TSM violates the fundamental principles of mathematics by way of one conspicuous example: the teaching of high school geometry. Since students are almost never given a definition of any concept in TSM (e.g., what is a "fraction"?), they come into their high school geometry course having no experience with using definitions for reasoning. Since TSM does not give students any hint of the *logical* structure of mathematics, they will have no conception of what "axioms" are before they run into them in high school geometry. Since TSM hardly ever exposes students to reasoning before high school geometry, "proving a theorem" will also be a completely foreign concept. Yet, in this geometry course, and *only* in this course in TSM, students are suddenly confronted with a litany of definitions, axioms, theorems and proofs. They are called upon to *prove every single assertion no matter how trivial*, and in fact, most of the early theorems are unbelievably trivial and therefore extremely difficult to "prove". Under these circumstances, geometry teaching and learning often degenerates into a farce, as is well-known.

Important as FPM may be to school mathematics, the mathematics we teach preservice teachers must do more than respect FPM. The mathematics we teach college math majors — groups, rings, fields, Dedekind cuts, Cauchy integral theorem, Gauss-Bonnet theorem — also respects FPM, but it is too advanced to be usable in the K–12 classroom. The content knowledge we require of our mathematics teachers therefore must meet *both* of the following conditions:

(I) It closely toes the line of the K–12 mathematics curriculum.

(II) It is consistent with FPM.

For brevity, we will refer to the body of mathematical knowledge that satisfies both (I) and (II) as **principles-based school mathematics (PBSM)**. TSM satisfies (I) but emphatically not (II), and the mathematics taught to college math majors satisfies (II) but definitely not (I). If we want both outcomes (A) and (B) to materialize in the preparation of American mathematics teachers, then the *"content knowledge"* in (A) must be understood to be PBSM. Indeed, if teachers only know TSM, then the upshot of any effort to foster lifelong learning and continuous education would likely result in the creation of a robust environment for TSM to thrive and even metastasize throughout the body of school mathematics education. We cannot import (A) and (B) into the American system without first getting rid of TSM.

In the context of PBSM, we can now raise a question concerning the preparation of Chinese teachers: in its emphasis on teachers' content knowledge (i.e., (A) above), does it also ensure that all Chinese teachers know some version of PBSM? We need an evidence-based answer to this question. My own interest in an answer is enhanced by my (limited) firsthand experiences . I have visited China (essentially only Beijing) eight times and — except for a one-week visit in 2010 — each visit lasted between three and seven weeks. In seven of the eight visits, I got to meet with officials from the Ministry of Education. Now, during my visits up to 1985 as a working mathematician, I never once suggested that I knew anything about school education, yet each time, those officials would ask me about what they called the "duck-stuffing" phenomenon (*tianya*, i.e., rote-memorization) in school mathematics classrooms. The fact that the same question — in the same terminology of *tianya* — was raised by different officials in the Ministry of Education suggests that maybe some version of TSM has also infected Chinese school mathematics education.

A second reason we cannot recommend (A) and (B) for the U.S. without qualification is the lack of time in the present American education system for teachers to engage in lifelong learning. If teachers are going to engage in mutual classroom visits, collaborative lesson planning, and attending professional development institutes as a way of life, they will have to *set aside a large block of time* in their job to engage in such time-consuming activities. It would appear that the Chinese education system allows its teachers to do that because Chinese teachers have much larger classes: typically around twice the size of U.S. classes. As a result, Chinese teachers have fewer classes than U.S. teachers, typically just two or three per day. For (B) to become a reality in the American system, teachers in the U.S. will have to make some hard choices. Are they willing to teach larger classes to make time for more opportunities for professional growth? If so, are they ready for the inevitable rise in discipline problems? In addition, such a sea change in the teaching profession will make a greater demand on each teacher's mathematical and pedagogical competence. This will require an upgrade of the teaching corps, and this upgrade will be imppossible unless we can make teaching a more attractive profession, such as giving teachers higher salaries and a greater input in administrative decisions. Do we have the political will to face up to this challenge?

Having stressed the importance of teaching PBSM rather than TSM in schools, we must point out that it is equally important to have available a default version of PBSM for the dual purpose of replacing inservice teachers' knowledge of TSM with PBSM and producing a new generation of teachers who are well-equipped with PBSM. In addition, there is another benefit of having available a default version of PBSM: it will facilitate the development of teachers' PCK. This is because PCK is the *bridge* between teachers' *content knowledge* and their *pedagogical practices* in the classroom. At the moment, the education literature seems to be uncertain about exactly what this bridge is, or might be. The common belief that PCK is needed to transform mathematics in "its academic form featuring rigorous deduction and logical reasoning" — which is "is precise but cold" — into something "interesting, beautiful, and easy to access" *and fun* in the school classroom speaks to the misconception about what constitutes the "content" in *pedagogical content knowledge* (PCK). If we correctly understand this "content" to be PBSM, then the fact that this content has already been conceived and developed in accordance with students' learning trajectories and the required curricular sequencing of topics (i.e., condition (I) of PBSM) means that it will be, by design, already much easier to access and present than the supposedly "precise but cold" mathematics.

Back in 1972, when Begle first looked into the disconnect between teachers' "mathematical" knowledge and the effectiveness of their teaching, he concluded:

> ... teachers should be provided with a solid understanding of the courses they are expected to teach.

Subsequent works on PD for teachers have basically borne out Begle's dictum. In our language, what Begle was saying is that all teachers should be taught PBSM. There is a tremendous amount of work ahead, but it needs to be done.