George M. Bergman
10 Evans Hall

Fall 1999, Math 1B

First Midterm

21 September, 1999
8:10-9:30 AM

1. (30 points, 6 points apiece) Find the following.

(a) $\int (x+1)\, e^x\, dx$

(b) $\int \sin^3 x\, \cos^3 x\, dx$

(c) $\int_{-1}^{1} \frac{(x+2)^2}{x^2+1}\, dx$

(d) An integral expressing the length L of the curve $y = \sin x$ from $x = a$ to $x = b$. Do not attempt to carry out the integration.

(e) $\lim_{n\to\infty} ((n + n^{-1})^2 - n^2)$

2. (40 points, 10 points apiece) Compute the following integrals.

(a) $\int \sin \sqrt{x}\, dx$

(b) $\int \frac{dx}{4\,x^{2/3} - 4\,x^{1/3} - 3}$

(c) $\int (6\,x - x^2)^{-1/2}\, dx$

(d) $\int_0^{e^{-2}} t^{-1}(\ln t)^{-2} dt$

3. (12 points) (a) (6 points) If f is a continuous function on the real line, what is meant by $\int_0^{\infty} f(x)\, dx$ (assuming this exists)?

(b) (6 points) Let f be a function such that $\int_0^{\infty} f(x)\, dx$ exists. Let us call its value L, and let c be any positive real number. Derive from the definition a formula expressing $\int_0^{\infty} f(cx)\, dx$ in terms of L and c. (Correct reasoning: 3 points; correct formula: 3 points.)

4. (18 points) (a) (9 points) State the Principle of Mathematical Induction.

(b) (9 points) Recall that the *Fibonacci numbers* are the sequence of numbers $f_1, f_2, f_3, \ldots$ defined by $f_1 = f_2 = 1$, and $f_{n+1} = f_n + f_{n-1}$ for $n \geq 2$. Prove that for all $n \geq 1$, $f_{n+1}^2 - f_{n+1} f_n - f_n^2 = (-1)^n$. (Suggestion: use Mathematical Induction. In proving S_{k+1} from S_k, apply the formula saying each Fibonacci number is the sum of the two preceding to the highest Fibonacci number occurring.)

P. Vojta

Math 1BM First Midterm

Thu 17 Feb 2000

1. (5 points) Estimate the integral $\int_0^\pi \sin x$ using Simpson's rule with $n = 6$.

2. (6 points) Find $\displaystyle\int_0^4 \frac{\ln x}{\sqrt{x}}\,dx$

3. (6 points) Find $\displaystyle\int_0^\infty \frac{\arctan x\,dx}{(1+x^2)\sqrt{(\pi/2)^2 - (\arctan x)^2}}$. Be careful to explain your steps.

4. (6 points) Determine whether the series $\displaystyle\sum_{n=1}^\infty \frac{3^n + 5^n}{15^n}$ converges or diverges. If it converges, find the sum.

5. (7 points) Find the area of the surface obtained by rotating the curve $y = e^x$, $0 \le x \le \ln 2$, about the x-axis.

6. (8 points) Find $\displaystyle\int \frac{5x^2 - 5x + 5}{x^3 - 2x^2 + x - 2}\,dx$.

7. (12 points) Find two of the following six integrals: [**Caution:** some of these integrals are *impossible!*]

(a).	$\displaystyle\int \frac{dx}{\ln x}$	(d).	$\displaystyle\int \frac{\sin^2 x}{\cos^4 x}\,dx$
(b).	$\displaystyle\int \sin\sqrt{x}\,dx$	(e).	$\displaystyle\int e^x \ln x\,dx$
(c).	$\displaystyle\int \sqrt[3]{x^2+1}\,dx$	(f).	$\displaystyle\int x\sqrt{x^2+4x+13}\,dx$

- Be sure to circle the ones you want credit for.
- "Divergent" is an acceptable answer (say why).
- "Impossible" is not an acceptable answer.
- Don't forget the formulas on page 2.
- Continue onto the other side of this sheet as necessary.

Department of Mathematics, University of California, Berkeley

Math 1B

Alan Weinstein, Spring 2002

First Midterm Exam, Thursday, February 21, 2002

Instructions. Be sure to write on the front cover of your blue book: (1) your name, (2) your Student ID Number, (3) your GSI's name (Tathagata Basak, Tameka Carter, Alex Diesl, Clifton Ealy, Peter Gerdes, John Goodrick, Matt Harvey, George Kirkup, Andreas Liu, Rob Myers, or Kei Nakamura).

Read the problems very carefully to be sure that you understand the statements. Show all your work as clearly as possible, and circle each final answer to each problem. **When doing a computation, don't put an "=" sign between things which are not equal. When giving explanations, write complete sentences. Remember: if we can't read it, we can't grade it.**

1 . [20 points] Evaluate each of the following.

(A)

$$\int e^{\sqrt{x}}\,dx$$

(B)

$$\int_{-\pi/3}^{0} \sin^2 x\,dx + \int_0^{\pi/3} \cos^2 x\,dx$$

(C)

$$\int_0^{\pi/2} \sin 2x \sin x\,dx$$

(D)

$$\int \frac{x^2+2}{x^2-x}\,dx$$

2 . [10 points] For which values of p is the integral

$$\int_0^{\pi} \frac{\sin x}{x^p}\,dx$$

improper? For which of these values of p is it convergent? You must justify your answers.

3 . [15 points] Suppose that a thin wire of uniform linear density λ (i.e. the mass of any segment of the wire is λ times its arc length) is represented by the graph of a function $y = f(x)$ defined in the interval $[a, b]$. Putting together what you know about arc length and centers of mass, write a formula for the mass m of the wire, and a pair of integral formulas for the coordinates $(\overline{x}, \overline{y})$ of the center of mass of the wire. The integrands in the formulas should be expressed in terms of $f(x)$ and, possibly, its derivative(s).

You must give some justification for your formulas (a limit argument or a "differential" argument), but it does not have to be a formal proof.

Mathematics 1B Professor K. A. Ribet Spring 1990

First Midterm Exam—60 points

1a *(5 points)*. Find $\lim_{t\to 0} \dfrac{\cos 3t - 1}{\cos 4t - 1}$.

1b *(7 points)*. Calculate $\displaystyle\int_2^4 \frac{\sqrt{16-x^2}}{x^2}\,dx$.

2a *(6 points)*. Find $\displaystyle\int \frac{\ln(x^2)}{x^2}\,dx$.

2b *(6 points)*. What approximation to $\displaystyle\int_0^6 (x^2-2x-6)\,dx$ is furnished by Simpson's Rule, when the interval $[0,6]$ is divided into 6 equal pieces?

[In problems 3–4, do *not* evaluate the integrals!]

3 *(8 points)*. The region between $y = \sin x$ and the x-axis, from $x = 0$ to $x = \pi/2$, is covered with a thin wafer weighing 20 pounds per unit area. Express as a definite integral the wafer's moment of inertia about the line $y = -3$.

4 *(6 points)*. A thin uniform wire weighing 300 tons is fitted over that part of the curve $y = x^3$ which runs from $x = 1$ to $x = 2$. Express in terms of definite integrals the x- and y-coordinates of the centroid of the wire.

5a *(5 points)*. Evaluate $\lim_{t\to 1} \dfrac{t-1}{\sqrt{t+1}-\sqrt{t-1}}$.

5b *(8 points)*. Find A, B, and C: $\dfrac{x^2-2x+4}{(x-1)(x^2-x+1)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2-x+1}$.

6 *(9 points)*. Calculate $\displaystyle\int_0^{1/2} \frac{8-16x}{8x^2-4x+1}\,dx$.

Math 1B Spring, 1994 Professor K. Ribet

First Midterm Exam—February 22, 1994

1a *(6 points).* Calculate $\displaystyle\int \frac{2u\,du}{u^2-2u+5}$.

1b *(3 points).* Write a definite integral which represents the length of the curve $y = \tan x$, $0 \le x \le \pi/4$.

2 *(7 points).* Find the area of the surface obtained by rotating the curve $y = \sin x$, $0 \le x \le 2\pi$ about the line $y = 0$.

Decide whether each of the following sequences converge or diverge. In the case of a convergent sequence, find the limit. Explain your reasoning!

3a *(4 points).* $a_n = \dfrac{3^n}{\pi^n}$;

3b *(4 points).* $b_n = \begin{cases} n\sin(1/n) & \text{if } n \text{ is odd} \\ (1+n)^{1/n} & \text{if } n \text{ is even;} \end{cases}$

3c *(4 points).* $c_n = \dfrac{1}{n^2} + \dfrac{2}{n^2} \cdots + \dfrac{n}{n^2}$.

4a *(7 points).* Evaluate $\displaystyle\int_0^1 x \sin^{-1} x\,dx$.

4b *(7 points).* Suppose that $f(0) = 3$. Use Simpson's rule with $n = 4$ to estimate $f(8)$:

x	0	2	4	6	8
$f'(x)$	2	4	1	3	5

5 *(7 points).* Write as a sum of partial fractions: $\dfrac{x^3+5x^2+2x+4}{x^4+2x^2}$.

6a *(6 points).* Evaluate the improper integral $\displaystyle\int_1^\infty \left(\frac{2x}{x^2+1} - \frac{2}{x+1}\right) dx$. (If the integral is divergent, answer "divergent" and explain your reasoning.)

6b *(5 points).* Find $\displaystyle\int_{\ln 2}^{\ln 3} \frac{dx}{e^x - 1}$.

Second midterm exam—April 7, 1994

1a *(4 points).* If the series $\displaystyle\sum_{n=1}^{\infty} (1-\sqrt{2})^n$ converges, calculate its value. If it is divergent, explain why.

Math 1B — First Midterm

V.Jones, Spring 1998

150 points total. The first 5 questions are Multiple Choice.
For each question mark an $\times$ in the most correct place
in the grid below. No partial credit for 1–5.
Questions 6 and 7 are not multiple choice.

	a	b	c	d	e
1					
2					
3					
4					
5					

MC ________

6 ________

7 ________

Math 1B Midterm

Multiple Choice Questions. Each multiple choice question worth 15 points.

1. Which of the following formulas is correct?

a) $\int u(x)v(x)dx = (\int u(x)dx)v(x) + u(x)\int v(x)dx$

b) $\int u(x)v(x)dx = \int u(x)dx + \int v(x)dx$

c) $u(x)v(x) = \int u(x)v'(x)dx + \int u'(x)v(x)dx$

d) $\int u(x)v(x)dx = \int u(x)v'(x)dx + \int u'(x)v(x)dx$

e) $\int u(x)v(x)dx = (\int u(x)dx)v'(x) + (\int v(x)dx)u'(x)$

2. If you wanted to expand

$$\frac{2x+7}{(x+1)^2(x^2+x+19)^2}$$

in partial fractions you would use the sum:

a) $\dfrac{A}{2x+7}+\dfrac{B}{(2x+7)^2}+\dfrac{C}{(2x+7)^3}+\dfrac{D}{(2x+7)^4}+\dfrac{E}{(2x+7)^5}$

b) $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{x^2+x+19}+\dfrac{Ex+F}{(x^2+x+19)^2}$

c) $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{x^2+x+19}+\dfrac{D}{(x^2+x+19)^2}$

d) $\dfrac{Ax+B}{(x+1)^2}+\dfrac{Cx+D}{(x^2+x+19)^2}$

e) $2x+7+\dfrac{A}{(x+1)^2}+\dfrac{B}{(x^2+x+19)^2}+\dfrac{2x+7}{x+1}+\dfrac{2x+7}{(x^2+x+19)^2}$

3. Which of the following functions **cannot** be integrated in terms of elementary functions?

a) $x^2 \ln x$

b) $x^2 e^{x^2}$

c) $e^x \sin x$

d) $\dfrac{1}{x \ln x}$

e) $x \sin(x^2)$

4. Which of the following statements is always correct for a function $f(x)$ with $0 \leq f(x) \leq C$?

   a) If $\int_1^\infty f(x)dx$ converges, so does $\int_1^\infty \sqrt{f(x)}dx$.

   b) If $\int_1^\infty f(x)dx$ converges, so does $\int_1^\infty f(x)^{-2}dx$.

   c) If $\int_1^\infty f(x)dx$ diverges, so does $\int_1^\infty \frac{f(x)}{\sqrt{x}}dx$.

   d) If $\int_1^\infty f(x)dx$ converges, so does $\int_1^\infty \frac{f(x)}{1+x}\,dx$.

   e) If $\int_1^\infty f(x)dx$ diverges, so does $\int_1^\infty f(x)^p dx$ for $p > 1$.

5. Which of the following statements is correct?

   a) The error bound for Simpson's rule is improved by a factor of 16 by doubling the number of points at which the function is evaluated.

   b) The trapezoid rule is exact for quadratic functions.

   c) There is no need to use Simpson's rule for any function involving sines, cosines and polynomials since it can always be integrated in terms of sines, cosines and polynomials.

   d) The error bound for the midpoint rule is $|E_M| \leq \dfrac{\max_{a\leq x\leq b}\left(|f^{(3)}(x)|\right)(b-a)^3}{24n^2}$

   e) Simpson's rule uses the best *linear* approximation to $f(x)$ on small intervals.

Not Multiple Choice

6. *(25 pts)* Find the arc-length function for the curve $y = \dfrac{x^2}{8} - \ln x$, starting at $(1, \frac{1}{8})$. (Evaluate the integral.)

7. Evaluate the following indefinite integrals:

   (i) *(10 pts)* $\displaystyle\int \frac{1}{(x+3)(x-2)}dx$

7.(ii) *(20 pts)* $\int (\ln x)^2 dx$

7.(iii) *(20 pts)* $\int \frac{1}{\sqrt{1-4x^2}}\, dx$