Math 1B, Fall 2008, Wilkening

Homework for Quiz 4 (on October 1)

1. Use the product rule to show that $\frac{d}{dx}x^n = nx^{n-1}$ for $n \geq 1$.
(Here $x^0 = 1$ even when $x = 0$.)

2. Suppose $f(x)$ is odd. Prove that $g(x) = f(x)^n$ is $\left\{ \begin{array}{c} \text{odd if } n \text{ is odd} \\ \text{even if } n \text{ is even} \end{array} \right\}$ for $n \geq 1$.

3. Use the fact that $(f+g)'(x) = f'(x) + g'(x)$ to prove that

$$\frac{d}{dx}\left(\sum_{i=1}^{n} f_i(x)\right) = \sum_{i=1}^{n} f_i'(x), \qquad (n \geq 1).$$

4. (#39 p. 193) Prove that $\lim_{x\to\infty} \frac{e^x}{x^n} = \infty$ for $n \geq 1$.

5. (#55 p. 196) Show that if $f(x) = xe^x$, then $f^{(n)}(x) = (x+n)e^x$ for $n \geq 1$.

6. (#52 p. 219) Prove that $e^x \geq 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$ for every real number $x \geq 0$ and integer $n \geq 1$.

7. Show that $2^n \geq n^2$ for $n \geq 4$.

8. (#31(c) p. 309) Show that for $n \geq 1$, $\int_0^{\pi/2} \sin^{2n+1} x\, dx = \frac{2 \cdot 4 \cdot 6 \cdot \cdots \cdot 2n}{3 \cdot 5 \cdot 7 \cdot \cdots \cdot (2n+1)}$

Sec. 8.1 #1,6,7,8,9,12,13,16,20,23,28,31,32,34,37,38,39,41,45,46