$$\frac{d}{dx}x^n = nx^{n-1}, \qquad \frac{d}{dx}e^x = e^x, \qquad \frac{d}{dx}\ln|x| = \frac{1}{x} \qquad (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$\sin^{-1}x = y \quad \Leftrightarrow \quad \sin y = x \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$
$$\cos^{-1}x = y \quad \Leftrightarrow \quad \cos y = x \quad \text{and} \quad 0 \le y \le \pi$$
$$\tan^{-1}x = y \quad \Leftrightarrow \quad \tan y = x \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

$$(fg)' = f'g + g'f, \qquad (f/g)' = \frac{gf' - fg'}{g^2} \qquad \frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

$$\sin^2 x + \cos^2 x = 1 \qquad \sin 2x = 2\sin x\,\cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\sec x = \sec x\,\tan x$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\int \tan u\,du = \ln\left|\sec u\right| + C \qquad \int \sec u\,du = \ln\left|\sec u + \tan u\right| + C$$

$$\int \tanh u\,du = \ln(\cosh u) + C \qquad \int \operatorname{sech} u\,du = \tan^{-1}\left|\sinh u\right| + C \qquad \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\sinh^{-1}x = \ln\left(x + \sqrt{x^2+1}\right) \qquad x \in \mathbb{R}$$
$$\cosh^{-1}x = \ln\left(x + \sqrt{x^2-1}\right) \qquad x \ge 1$$
$$\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -1 < x < 1$$

$$\frac{d}{dx}\sinh^{-1}x = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}\tanh^{-1}x = \frac{1}{1-x^2}$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \cosh 2x = \cosh^2 x + \sinh^2 x \qquad \sinh 2x = 2\sinh x\,\cosh x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2} \qquad \sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$\frac{d}{dx}\sinh x = \cosh x \qquad \frac{d}{dx}\cosh x = \sinh x \qquad \frac{d}{dx}\tanh x = \operatorname{sech}^2 x \qquad \frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$$

Newton's method for solving $f(x) = 0$: $\quad x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \qquad \int u\,dv = uv - \int v\,du$$

MVT: if f is continuous on $[a,b]$ and differentiable on (a,b), there is a $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

$$L_n = \sum_{i=0}^{n-1} f(x_i)\Delta x, \qquad R_n = \sum_{i=1}^{n} f(x_i)\Delta x \qquad S_n = \Big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big]\frac{\Delta x}{3}$$

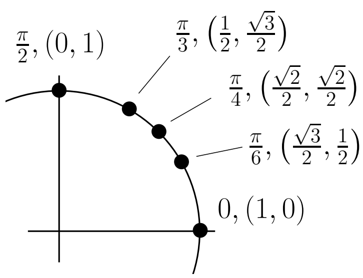
$$M_n = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x, \quad \Delta x = \frac{b-a}{n}, \quad \bar{x}_i = \frac{x_{i-1}+x_i}{2} \qquad T_n = \Big[\tfrac{1}{2}f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \tfrac{1}{2}f(x_n)\Big]\Delta x$$

$$E_T = \int_a^b f(x)\,dx - T_n, \quad |E_L| \le \tfrac{K_1(b-a)^2}{2n}, \quad |E_R| \le \tfrac{K_1(b-a)^2}{2n}, \quad |E_T| \le \tfrac{K_2(b-a)^3}{12n^2}, \quad |E_M| \le \tfrac{K_2(b-a)^3}{24n^2}, \quad |E_S| \le \tfrac{K_4(b-a)^5}{180n^4} \quad K_j = \max_x |f^{(j)}(x)|$$

Comparison theorem: Suppose $f(x) \ge g(x) \ge 0$ for $x \in (a,b)$.
(1) if $\int_a^b f(x)\,dx$ is convergent, then $\int_a^b g(x)\,dx$ is convergent.
(2) if $\int_a^b g(x)\,dx$ is divergent, then $\int_a^b f(x)\,dx$ is divergent.
($a = -\infty$ and/or $b = \infty$ are allowed.)

$\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ is convergent if $p > 1$ and divergent if $p \le 1$.

$\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ is convergent if $p < 1$ and divergent if $p \ge 1$.



Partial Fractions:

$$\frac{3x^5 + 2x + 7}{(x^2+x+1)^2 x^3 (x-2)} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2} + \frac{E}{x} + \frac{F}{x^2} + \frac{G}{x^3} + \frac{H}{x-2}$$

Trig Substitution

$\sqrt{a^2-x^2}$	$x = a\sin\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$ or $x = a\sinh\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$ or $x = \pm a\cosh\theta$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots, \ (R = \infty) \qquad \tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots, \ (R = 1)$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots, \ (R = \infty)$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots, \ (R = \infty)$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots, \ (R = 1)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots, \ (R = 1)$$

Squeeze theorem: if $a_n \le b_n \le c_n$ for $n \ge N$ and $a_n \to L$ and $c_n \to L$ then $b_n \to L$.

If $\lim_{n\to\infty} |a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$. If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

Integral test: if $a_n = f(n)$ for $n \ge N$ and $f(x)$ is continuous, positive and decreasing for $x \ge N$, then ($\sum a_n$ is convergent) $\Leftrightarrow$ (there is an x_0 such that $\int_{x_0}^{\infty} f(x)\,dx$ is convergent)

Comparison test: Suppose $0 \le a_n \le b_n$ for $n \ge N$. Then
(1) $\sum b_n$ convergent $\Rightarrow$ $\sum a_n$ convergent. (2) $\sum a_n$ divergent $\Rightarrow$ $\sum b_n$ divergent.

Limit comparison test: Consider the series $\sum a_n$ and $\sum b_n$. Suppose $b_n > 0$ for $n \ge N$.
If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where $0 < c < \infty$, then either both series converge or both diverge.

Alt. series: if $0 \le b_{n+1} \le b_n$ for $n \ge 1$ and $b_n \to 0$ as $n \to \infty$, then $\sum (-1)^n b_n$ converges.
estimation: $|s - s_n| < b_{n+1}$, where $s = \sum_{n=1}^{\infty} (-1)^n b_n$ and $s_n = \sum_{i=1}^{n} (-1)^i b_i$.

Ratio test: Suppose $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$. Root test: Suppose $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$.
if $L < 1$: absolutely convergent. if $L > 1$ divergent. if $L = 1$ test is inconclusive.

Radius of convergence: $\sum_{n=0}^{\infty} c_n (x-a)^n$ converges for $|x-a| < R$, diverges for $|x-a| > R$

If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ has a positive radius of convergence, then $c_n = \frac{f^{(n)}(a)}{n!}$.

Taylor's formula: $f(x) = \left[f(a) + f'(a)(x-a) + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n \right] + R_n(x)$,
where $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}$ for some z between a and x.

arclength: $L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$, $s(x) = \int_a^x \sqrt{1 + f'(t)^2}\,dt$, $ds = \sqrt{dx^2 + dy^2}$.

work: $W = \int F\,dx$ or $W = \int dW$, $dW =$ (distance moved)$\times$(force on slice or segment)

spring: $F = k(x - x_0)$, $x_0 =$ natural length, $k =$ spring constant

hydrostatic pressure: $P = \rho g d$, hydro. force: $F = \int P\,dA$, $dA =$ area of horizontal slice
$\rho =$ density (kg/m^3), $g =$ gravitational acceleration (m/s^2), $d =$ depth (m)

moments: $M_y = \sum_i m_i x_i$, $M_x = \sum_i m_i y_i$, center of mass: $\bar{x} = \frac{M_y}{m}$, $\bar{y} = \frac{M_x}{m}$, ($m = \sum_i m_i$)

centroid: $\left\{ \begin{array}{l} \bar{x} = \frac{1}{A}\int_a^b x[f(x) - g(x)]dx = \frac{1}{A}\int_c^d \frac{1}{2}[F(y)^2 - G(y)^2]dy \\ \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)^2 - g(x)^2]dx = \frac{1}{A}\int_c^d y[F(y) - G(y)]dy \end{array} \right\}$. Pappus: $V = 2\pi \bar{x} A$

separation of variables: $y' = f(x)g(y)$, $\int \frac{dy}{g(y)} = \int f(x)\,dx$.

mixing problems: $\frac{dy}{dt} =$ (rate in) $-$ (rate out), $y =$ amount in container (e.g. kg of salt)

2nd order homogeneous: $ay'' + by' + cy = 0$. (spring: $a \to m$, $b \to c$, $c \to k$, $y \to x$, $x \to t$)

aux. eqn: $ar^2 + br + c = 0$. $y(x) = \begin{cases} c_1 e^{r_1 x} + c_2 e^{r_2 x}, & b^2 - 4ac > 0 \\ c_1 e^{rx} + c_2 x e^{rx}, & b^2 - 4ac = 0 \\ e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x), & b^2 - 4ac < 0,\ r = \alpha \pm i\beta \end{cases}$

nonhomogeneous problem: $ay'' + by' + cy = G(x)$. Method of undetermined coefficients: if $G(x) = P(x)e^{kx}\cos(mx)$ or $G(x) = P(x)e^{kx}\sin(mx)$, try $y_p(x) = Q(x)e^{kx}\cos(mx) + R(x)e^{kx}\sin(mx)$ with $Q(x)$, $R(x)$ of the same degree as $P(x)$. If $(k + im)$ is a root of the auxiliary equation, multiply by x. If $m = 0$ and k is a double root, multiply by x^2.

variation of parameters: $y = u_1 y_1 + u_2 y_2$, $\begin{array}{l} y_1 u_1' + y_2 u_2' = 0, \\ y_1' u_1' + y_2' u_2' = G/a \end{array}$

Series solutions: substitute $y = \sum_{n=0}^{\infty} c_n x^n$ into equation. Relabel indices to get x^n. Peel off leading terms so sums start in same place. Match terms, set coefficients of x^0, x^1, ... to zero. Change indices to express recurrence as $c_n = ...$, where ... involves earlier coefficients. If $y(0)$ or $y'(0)$ are given, compute c_0 and c_1. Make table of first several coefficients. Try to recognize general formula. Assemble the solution using e.g. $y = \sum_{n=0}^{\infty} [c_{2n} x^{2n} + c_{2n+1} x^{2n+1}]$ or $y = \sum_{n=0}^{\infty} [c_{3n} x^{3n} + c_{3n+1} x^{3n+1} + c_{3n+2} x^{3n+2}]$.

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \cdots, \ (R = 1), \qquad \binom{k}{n} = \frac{k(k-1)\cdots(k-n+1)}{n!}, \qquad \binom{k}{0} = 1$$