

Midterm 1, Version A

Please write your name, section number, GSI's name, and version of your exam on your blue book, and write your name on your sheet of notes.

1. (2 points) Interpret the following limit as a derivative and evaluate the limit using the appropriate differentiation rule.

$$\lim_{x \rightarrow 1} \frac{x^{50} - 1}{x - 1}$$

2. (4 points) Compute the following derivative:

$$\frac{d}{dx} \bigg|_{x=1} \frac{x^{7/3}}{x^{1/3} + x^{4/3}}$$

3. (4 points) Suppose f is even and g is odd. Determine whether each of the following functions is even, odd or neither. If the function is even or odd, prove it. If neither, give an example of f and g such that the new function is neither even nor odd.

$$(i) f \circ g \quad (ii) g \circ f \quad (iii) fg \quad (iv) f + g$$

4. (5 points) Let $f(x) = x \cos x$. Show that there is a point c between 0 and $\pi/2$ such that the tangent line to the curve at the point $(c, f(c))$ is horizontal.

5. (5 points) Suppose $\lim_{x \rightarrow \infty} [f(x) - 2x] = 3$. Evaluate $\lim_{x \rightarrow \infty} \frac{f(x)}{x}$. Justify your answer.

6. (5 points) Evaluate the limit

$$\lim_{t \rightarrow 0} \frac{t^2}{1 - \cos 5t}$$

7. (5 points) Use the δ - ε definition of the limit to prove that $\lim_{x \rightarrow 1} \sqrt{x} = 1$.

Midterm 1, Version B

Please write your name, section number, GSI's name, and version of your exam on your blue book, and write your name on your sheet of notes.

1. (2 points) Interpret the following limit as a derivative and evaluate the limit using the appropriate differentiation rule.

$$\lim_{x \rightarrow 1} \frac{x^{75} - 1}{x - 1}$$

2. (4 points) Compute the following derivative:

$$\frac{d}{dx} \bigg|_{x=1} \frac{x^{4/3}}{x^{1/3} + x^{7/3}}$$

3. (4 points) Suppose f is even and g is odd. Determine whether each of the following functions is even, odd or neither. If the function is even or odd, prove it. If neither, give an example of f and g such that the new function is neither even nor odd.

$$(i) f \circ g \quad (ii) g \circ f \quad (iii) fg \quad (iv) f + g$$

4. (5 points) Let $f(x) = x \cos x$. Show that there is a point c between 0 and $\pi/2$ such that the tangent line to the curve at the point $(c, f(c))$ is horizontal.

5. (5 points) Suppose $\lim_{x \rightarrow \infty} [f(x) - 4x] = 3$. Evaluate $\lim_{x \rightarrow \infty} \frac{f(x)}{x}$. Justify your answer.

6. (5 points) Evaluate the limit

$$\lim_{t \rightarrow 0} \frac{t^2}{1 - \cos 3t}$$

7. (5 points) Use the δ - ε definition of the limit to prove that $\lim_{x \rightarrow 1} \sqrt{x} = 1$.