Math 1A, Spring 2008, Wilkening

## Another Sample Midterm 2

1. (1 point) write your name, section number, and GSI's name on your exam and write your name on your sheet of notes.
2. (3 points) Suppose $f$ is twice differentiable on the interval $[0,4]$ and satisfies

$$
\begin{aligned}
f^{\prime}(0) & =1 & f^{\prime}(1) & =0 & f^{\prime}(2) & =0 \\
f^{\prime \prime}(0) & =-1 & f^{\prime \prime}(1) & =-2 & f^{\prime \prime}(2) & =0
\end{aligned}
$$

At the endpoints $x=0$ and $x=4$, these are one-sided derivatives. Fill in the following table with YES, NO, or CBT (cannot be determined).

| $c=$ | 0 | 1 | 2 | 3 | 4 |
| ---: | ---: | ---: | ---: | :---: | :---: |
| $f$ has a local max at $c$ |  |  |  |  |  |
| $f$ has a local min at $c$ |  |  |  |  |  |

3. (5 points) Let $f(x)=x^{x}$. Compute $f^{\prime}(2), f^{\prime}(4)$ and $(f \circ f)^{\prime}(2)$. Note that $4^{4}=256$.
4. (5 points) Use a linear approximation to estimate: $\frac{1}{\pi} \tan ^{-1}\left(1+\frac{\pi}{100}\right)$.
5. (6 points) Two carts are connected by a 35 foot rope that passes over a pulley 12 feet above the floor. Cart A is being pulled to the left at a speed of $2 \mathrm{ft} / \mathrm{sec}$. How fast is cart B moving at the instant cart A is 9 feet from the point on the floor beneath the pulley?

6. (5 points) Show that there is exactly one $x \in \mathbb{R}$ satisfying

$$
x^{5}+e^{x}-2=0 .
$$

7. (5 points) Do one of the following:
(a) Show that

$$
\tanh \left(\sinh ^{-1} x\right)=\frac{x}{\sqrt{1+x^{2}}} \quad(x \in \mathbb{R})
$$

(b) If $g(x)=1+x+e^{x}$, find $g^{-1}(2)$ and $\left(g^{-1}\right)^{\prime}(2)$.

