Sample Midterm 1

You are allowed one 8.5×11 sheet of notes, one side only. This sheet must be turned in with your exam. *Calculators are not allowed.*

1. (2 points) What is the domain of the function $f(x) = \dfrac{\sqrt{x-2} + \sqrt{7-x}}{x-5}$?

2. (3 points) Compute the following derivative:

$$\frac{d}{dx}\bigg|_{\theta=0} \tan\theta \, \cos\theta$$

3. (5 points) Find the equation of the tangent line to the curve $y = 1/(1+x^2)$ passing through the point $\left(3, \frac{1}{10}\right)$.

4. (5 points) Compute $\displaystyle\lim_{x\to\infty} \frac{(x-1)(2x-3)}{(4x+1)(7x+1)}$

5. (5 points) Prove that $f(x) = 1 - x^5$ has a fixed point (i.e. there is a number c such that $f(c) = c$).

6. (5 points) Evaluate the limit

$$\lim_{\theta\to 0^-} \frac{\sin\theta}{\sqrt{1 - \cos\theta}}$$

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7. (5 points) Use the δ-ε definition of the limit to prove **one** of the following:

(a) $\displaystyle\lim_{x\to 2} 1/x = 1/2$.

(b) Suppose $\displaystyle\lim_{x\to 0} f(x) = L$. Define $g(x) = f(-x)$. Then $\displaystyle\lim_{x\to 0} g(x) = L$.

Another Sample Midterm 1

1. (2 points) Determine whether f is even, odd or neither: $f(x) = \dfrac{x}{1+x^2}$.

2. (3 points) Let $f(x) = \dfrac{x^{3/2} + 2\sqrt{x}}{x^5}$. Evaluate $f'(1)$.

3. (5 points) Find all the points on the curve $y = x^3 + 3x^2 + 3x + 1$ where the tangent line is horizontal.

4. (5 points) Find the value a such that the following limit exists and evaluate the limit:
$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$

5. (5 points) Compute $\lim_{x \to \infty} x \tan\left(\dfrac{\pi}{x}\right)$. Hint: $\lim_{x \to \infty} f(x) = \lim_{x \to 0^+} f(1/x)$.

6. (5 points) Evaluate the limit
$$\lim_{x \to -\infty} \left(\sqrt{x^2 + x + 1} - \sqrt{x^2 + 1} \right)$$

7. (5 points) A stone is dropped from height H on a planet with gravitational constant g. The equation of motion of the stone is $y(t) = H - \frac{1}{2}gt^2$. Show that the instantaneous velocity of the stone when it hits the ground is twice the average velocity during its fall.