Mathematics 53

Quiz 3 – 07/16

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This is a closed book/notes test. Calculators are not permitted

1. Find the tangential and normal components of the acceleration vector of a particle with position function $\mathbf{r}(t) = (t, 2t, t^2)$.

2. Find the curvature of the ellipse $x = 3\cos t$, $y = \sin t$ at the points $(3, 0)$ and $(0, 1)$.

3. Find the length of the curve $\mathbf{r}(t) = (t^{\frac{3}{2}}, \cos(2t), \sin(2t))$ for $0 \leq t \leq 1$.

4. Show that the curve with vector equation

$$\mathbf{r}(t) = (a_1t^2 + b_1t + c_1, a_2t^2 + b_2t + c_2, a_3t^2 + b_3t + c_3)$$

lies in a plane and find an equation of the plane.

5. (extra credit!) In class we derived the following formulae for Frenet-triple $\mathbf{T}, \mathbf{N}, \mathbf{B} = \mathbf{T} \times \mathbf{N}$

$$\frac{d\mathbf{T}}{ds} = \kappa \mathbf{N}\,,$$
$$\frac{d\mathbf{N}}{ds} = -\kappa \mathbf{T} + \tau \mathbf{B}\,,$$
$$\frac{d\mathbf{B}}{ds} = -\tau \mathbf{N}\,,$$

Using these relations show that

(a) $\mathbf{r}'' = s''\mathbf{T} + \kappa (s')^2 \mathbf{N}\,,$

(b) $\mathbf{r}' \times \mathbf{r}'' = \kappa (s')^3 \mathbf{B}\,,$

where primes denote derivatives with respect to t and s is the arc-length parameter.