MATH 53

Midterm – 07/23

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This is a closed book/notes test. Calculators are not permitted

1. Consider the curve in $\mathbb{R}^2$ defined by the parametric quations

$$x = e^t, \qquad y = e^{2t} - 2e^t + 1\,.$$

Write down the Cartesian equation of this curve, sketch the curve, and indicate with an arrow the direction in which the curve is traced as the parameter t is increasing.

2. Sketch the curve $r = 3 + 3\cos\theta$ and find the area enclosed by this curve.

3. Find an equation of the plane containing the line

$$\frac{x-1}{2} = \frac{y+2}{3} = -z$$

and the point $(-2, 0, 5)$.

4. Find parametric equations of the line of intersection of the planes $3x - 2y + z = 1$ and $2x + y - 3z = 3$.

5. Find an equation for the surface consisting of all points P in the three-dimensional space such that the distance from P to the point $(0, -1, 0)$ is equal to the distance from P to the plane $y = 1$.

   Identify this surface by name and sketch it.

6. Find the differential of the function $f(x, y, z) = \sqrt{x^2 + 4y^2 + z^2}$ and use it to approximate the number $f(1.98, 1.01, 1.02)$.

7. Write down an equation of the tangent plane to the surface $y = x^2 z - 2xz^3 + z^2$ and the point $(2, 1, 1)$.

8. Let $f(x, y)$ be a function with continuous second partial derivatives. Suppose that $x = au + bv$ and $y = -bu + av$, where a and b are two real numbers such that $a^2 + b^2 = 1$. Show that

$$\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

9. Show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^4 y^3 \sin x \cdot \cos y}{x^{10} + y^6}$$

does not exist.