The Midterm will cover all assigned sections from Chapters 1 and 2. You should know all the definitions and theorems we have covered in lecture. Knowing a definition means you should be able to state it precisely, and you should know how to go about checking whether a given object satisfies the definition or not. You should be aware of what *type* of object is appropriate whenever you use any of these terms.

Key definitions:

- linear system, existence, uniqueness,consistency, solution space.
- augmented matrix, row operations, row reduction, pivots, REF, RREF.
- pivot variables and free variables, general solution.
- vectors, $\mathbb{R}^n$, linear combinations, span.
- matrix vector product and relation to linear combinations.
- homogeneous and inhomogeneous systems.
- linear dependence and independence.
- linear transformations, 1-1 and onto functions.
- matrix transformations, standard matrix.
- matrix multiplication, composition of linear transformations.
- inverses, invertibility in terms of 1-1 onto, linear independence and span, and pivots.
- linear subspaces, column space and null space.
- coordinates with respect to a basis.
- dimension of a subspace.

Key Theorems:

The theorems we have covered describe how the definitions above are related to each other. The beauty and power of linear algebra comes from the fact that there are lots and lots of connections and they fit together very nicely. You should know the statements all the theorems stated in the lecture notes and in the assigned sections of the textbook (if I stated some theorem slightly differently than the book, either version is acceptable).

Some of the theorems just state basic properties of the definitions, which are crucial to know. Some of them are deeper and more surprising, and it is important to understand their proofs — without understanding the proofs, you will not understand *why* the connections work, and how to use them. The most important theorems in this regard are: Ch1: Theorems 4*, 7*, 8*, 10, 11*, 12*; Ch2: Theorems 5, 7, 8*, 9, 12, 13, 14. I may ask you to prove the theorems that are starred.

On the midterm, I may ask you to give a proof of one of these theorems. The proof does not have to be entirely formal or identical to the one in the book, but I want a clear explanation of why the theorem is true.

Algorithms. There is only one algorithm so far in this course: Row Reduction. This algorithm is used both to solve concrete problems, as well as to prove theorems (by reasoning about pivots).

Types of Problems. The problems will be similar to the homework problems, but to help you study, here are the main kinds:

- Given a linear system: determine whether it is consistent, find its general solution, reduce it to REF or RREF, determine whether it is consistent for all right hand sides.
- Given a set of vectors: determine their span, whether they are linearly independent, whether another vector is in the span. Find a basis for the span.
- Given a matrix: determine its inverse, multiply it by another matrix, determine whether its columns are linearly independent/span. Find bases for its column space and null space. Determine the dimension of its column space and null space.
- Given a linear transformation: write its standard matrix, determine whether it is 1-1/onto, find its inverse or composition with another linear transformation.
- Determine whether a given transformation is linear, whether a given set is a linear subspace.
- Give an example of a matrix/vector/transformation with a specified property.
- Variations on and combinations of all of the above. I will often give you a problem that requires you to solve several of the above as subproblems.
- State a theorem from class and give a proof of it.
- True/False questions.