Math 121A Spring 2015, Sample Midterm 1

Do 4 out of 5 of the following questions (indicate which ones).

1. Using appropriate tests, determine whether the following series diverge or converge:

$$\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2+3}, \qquad \sum_{n=1}^{\infty} \frac{2^n}{n!}.$$

2. Write down the first four coefficients of the Maclaurin series for

$$\frac{\log(1+x)}{1+x+x^2} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

3. Compute the eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

   Is it diagonalizable? Find $\mathrm{tr}(A^{11})$, where $\mathrm{tr}(M) = \sum_{i=1}^{n} M_{ii}$ denotes the trace of an $n \times n$ matrix.

4. Suppose $x^2 s + y^2 t = 1$ and $x + y = st$. Find

$$\left(\frac{\partial x}{\partial s}\right)_t \qquad \text{and} \qquad \left(\frac{\partial x}{\partial t}\right)_s,$$

   as functions of x, y, s, t.

5. Using Lagrange multipliers, find the largest box (in volume, with sides parallel to the coordinate axes and with center at the origin) which can be inscribed in the ellipsoid:

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1.$$