Math 121A Spring 2015, Extra Linear Algebra Problems

1. Consider the rotation matrix $[R_\theta]$ in the standard basis, from HW3. Write $[R_\theta]_B$ for the basis $B = \{(1,1)^T, (0,1)^T\}$. Is $[R_\theta]_B$ an orthogonal matrix?

   Now try it again for $[R_\theta]_C$ where $C = \{(1,1)^T, (1,2)^T\}$. Is $[R_\theta]_B = [R_\theta]_C$?

2. Boas Section 3.11: 27, 29.

3. The tribonacci sequence is defined by

   $$f_0 = 1, f_1 = 3,$$

   and

   $$f_{n+1} = \frac{f_n}{2} + 3f_{n-1}.$$

   Use diagonalization to derive an expression for f_n. Does it grow faster or more slowly than the Fibonacci sequence?

4. Consider the real vector space of polynomials in two variables x, y of degree at most 1 in each variable:

   $$V = \{a_0 + a_{10}x + a_{01}y + a_{11}xy : a_0, a_{01}, a_{10}, a_{11} \in \mathbb{R}\}.$$

   Consider the partial differentiation operator $T : V \to V$ given by:

   $$T(p(x,y)) = \frac{\partial p}{\partial y}(x,y),$$

   Write down the matrix of T in the basis $\{1, x, y, xy\}$ (i.e., use this as both the input basis and the output basis).

   Bonus: Repeat the exercise for the operator which differentiaties with respect to x:

   $$S(p(x,y)) = \frac{\partial p}{\partial x}(x,y).$$

   Do the matrices of S and T commute?

5. Work out the oscillator example from class (page 165, figure 12.1) with three unit masses and four springs of spring constant k.