Math 53 Homework # 7, due 3/5/99

1. E&P section 6.4 problem 1. (Ignore the "almost linear" part.)

2. Consider the system $x' = x + y$, $y' = 1 - x^2$.

   (a) Find the critical points, and determine their type and stability.

   (b) For each saddle point, find the directions of the incoming and outgoing trajectories. For each spiral point, determine if the trajectories move clockwise or counterclockwise.

   (c) Sketch a phase portrait.

3. Show that a critical point of $x' = F(x, y), y' = G(x, y)$ with linearization A is guaranteed to be asymptotically stable if $\mathrm{Tr}(A) < 0$ and $\det(A) > 0$. (Here 'Tr' denotes trace. Recall that the trace is the sum of the eigenvalues and the determinant is their product.)

4. Consider the following predator-prey model (compare E&P, p. 359):

$$\begin{aligned} x' &= kx\left(1 - \frac{x}{M}\right) - bxy, \\ y' &= -cy + dxy. \end{aligned}$$

   Here b, c, d, k, M are positive constants. Assume that $M > c/d$.

   (a) Show that there exists a unique critical point with $x, y > 0$.

   (b) Show that this critical point is asymptotically stable. (Problem 3 may be helpful here.)

5. Recall that a second order differential equation is equivalent to a system of two first order equations. (See E&P, p. 222.)

   (a) Do E&P section 6.4 problem 11.

   (b) Show that the roots r of the polynomial associated to a second-order constant coefficient linear equation $x'' + ax' + bx = 0$ are the same as the eigenvalues of the linearization of the critical point $(0, 0)$ of the equivalent first-order system.