Math 53 Homework # 6, due 2/26/99

1. Prove that if λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2.

2. Suppose an $n \times n$ matrix A has n linearly independent eigenvectors $v_1, \ldots, v_n$, with eigenvalues $\lambda_1, \ldots, \lambda_n$. Let S be the $n \times n$ matrix whose columns are $v_1, \ldots, v_n$, and let Λ be the diagonal $n \times n$ matrix whose diagonal entries are $\lambda_1, \ldots, \lambda_n$.

   (a) Prove that
   $$A = S\Lambda S^{-1}.$$
   (Factoring A in this way is called "diagonalizing" A.) Hint: first show that $(S^{-1}AS)e_i = \Lambda e_i$, where e_i is the standard basis vector whose i^{th} component equals 1 and whose other components are zero. Deduce that $S^{-1}AS = \Lambda$, and then that $A = S\Lambda S^{-1}$.

   (b) Using (a), show that if k is a positive integer then $A^k = S\Lambda^k S^{-1}$.

   (c) Diagonalize $A = \begin{pmatrix} -13 & -5 \\ 34 & 13 \end{pmatrix}$. (You may need complex numbers.)

   (d) Using (b), calculate the 1999^{th} power of the above matrix A.

3. Find the general solution to $\mathbf{x}' = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix} \mathbf{x}$.

4. Solve $\mathbf{x}' = \begin{pmatrix} -7 & 4 \\ -12 & 7 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 3+t \\ 5+t \end{pmatrix}$, with initial condition $\mathbf{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

5. E&P section 6.1 problems 4,6,30.

6. E&P section 6.2 problems 8,34.

7. How long did this assigment take? What was most difficult?