Math 53 Homework # 1, due 1/15/99

For each problem, try to write a coherent explanation of how you obtain the answer. It is not necessary to show every tiny algebraic step, although you should show the big steps. See the examples in the book.

1. (E&P 1.1: 36) Write down a differential equation that is a mathematical model of the following situation: in a city with a fixed population of P persons, the time rate of change of the number N of persons infected with a certain contagious disease is proportional to the product of the number who have the disease and the number who do not.

2. (E&P 1.2: 8; 1.4: 10, 22; 1.5: 10, 20; 1.6: 20) Find explicit solutions to the following differential equations. Here y' means $\frac{dy}{dx}$.

   (a) $y' = \cos 2x; \quad y(0) = 1.$
   (b) $(1 + x^2)y' = (1 + y)^2.$
   (c) $y' = 4x^3y - y, \quad y(1) = -3.$
   (d) $2xy' - 3y = 9x^3.$
   (e) $y' = 1 + x + y + xy, \quad y(0) = 0.$
   (f) $y^2y' + 2xy^3 = 6x$. (Hint: make a clever substitution.)

3. Suppose we know that $\frac{dy}{dx} = \sqrt{|y|}$ and $y(0) = 0$. Is $y(1)$ uniquely determined? Explain.

4. (E&P 1.4: 27) A certain city had a population of 25,000 in 1960 and a population of 30,000 in 1970. Assume that its population will continue to grow exponentially at a constant rate. What population can its city planners expect in the year 2000?

5. (E&P 1.2: 19) The brakes of a car are applied when it is moving at 100 km/h and provide a constant deceleration of 10 meters per second per second (m/s^2). How far does the car travel before coming to a stop?

6. (Extra credit challenge problem) Suppose $\frac{dy}{dx} = y$ and $y(0) = 0$. Prove that $y(x)$ *must* equal zero for all x. (Give a self-contained explanation without appealing to theorems stated in the book.)

7. How much time did you spend on this problem set? Which parts were most difficult?