

### Check your understanding

3. Which of the following shapes is the best approximation to the curve parametrized by  $x = (\cos t)^{1/101}$ ,  $y = (\sin t)^{1/101}$ ,  $0 \leq t \leq 2\pi$ ?
- (a) The square with vertices  $(\pm 1, \pm 1)$ .
  - (b) The square with vertices  $(\pm 1, 0)$  and  $(0, \pm 1)$ .
  - (c) The “plus sign” consisting of the line segment from  $(-1, 0)$  to  $(1, 0)$  and the line segment from  $(0, -1)$  to  $(0, 1)$ .

Answer: (a)

Explanation: Eliminating the  $t$  variable gives the equation  $x^{202} + y^{202} = 1$ . This curve is close to the square in (a), roughly because for a point on the square one has  $x^{202} + y^{202} \geq 1$ ; and as one moves towards the origin,  $x^{202} + y^{202}$  rapidly decreases, so there is a nearby point where  $x^{202} + y^{202} = 1$ . Note that the curve  $x = (\cos t)^{101}$ ,  $y = (\sin t)^{101}$ ,  $0 \leq t \leq 2\pi$  is close to the “plus sign” in (c).