Math 53 Midterm #2, 11/13/03, 8:10 AM – 9:30 AM

No calculators or notes are permitted. Each of the 6 questions is worth 10 points. Please write your solution to each of the 6 questions on a separate sheet of paper with your name and your TA's name on it. Please put a box around the final answer. To maximize credit, please show your work, and use extra time to double check that you got the correct answer and didn't misunderstand the question. Good luck!

General hint: if something is hard to calculate, try using something you've learned in order to calculate it in a different and easier way.

1. Find the minimum and maximum values of the function
$$f = (x-1)^2 + (y-1)^2$$
on the unit disc $x^2 + y^2 \le 1$.

2. Calculate
$$\int_0^1 \int_{x^{2/3}}^1 x\cos(y^4)\,dy\,dx.$$

3. Calculate
$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)^{2003}\,dx\,dy.$$

4. Find the area of the region enclosed by the curve
$$x^2 + xy + y^2 = 1.$$
Hint: use the substitution
$$x = u + v\sqrt{3}, \qquad y = u - v\sqrt{3}.$$

5. Let C be a plane curve starting at $(0,0)$ and ending at $(1,1)$. Let
$$\mathbf{F} = \left\langle x^2 + y, y^2 + x \right\rangle.$$
   (a) Show that $\int_C \mathbf{F} \cdot d\mathbf{r}$ has the same value for every C as above.
   (b) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for a curve C as above.

6. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the unit circle oriented counterclockwise, and $\mathbf{F}$ is the following vector field in the plane:
$$\mathbf{F} = \left\langle -y^3 + \sin(\sin x), x^3 + \sin(\sin y) \right\rangle.$$