Math 53 Final, 12/15/07, 12:30 PM – 3:30 PM

No electronic devices or notes are permitted. Each question is worth 10 points. Please write your solution to each of the 10 questions on a separate sheet with your name, SID#, and GSI on it. (If you are removing an incomplete for professor X, write "Incomplete/X" on each page. For Math 49, do questions 5-10 only.) To get full credit for a question, you must obtain the correct answer, put a box around it, and show correct work/justification. Please do not leave the exam between 3:20 and 3:30. Good luck!

1. Consider the space curve

$$\mathbf{r}(t) = \langle 1, t, t^2 \rangle.$$

Let f be a function on $\mathbb{R}^3$ satisfying $\nabla f(1, 2, 4) = \langle 5, 6, 7 \rangle$. Calculate

$$\frac{d}{dt} f(\mathbf{r}(t))\Big|_{t=2}.$$

2. Find the volume of the solid region consisting of all points that are above the surface $z = x^2$ and below the surface $z = 4 - y^2$.

3. Find the minimum distance from a point on the surface

$$z = x^2 + y^2 - 3/2$$

to the origin. Describe the set of points on the surface that are closest to the origin. (You may take it for granted that such points exist.)

4. Calculate the double integral

$$\iint_D xy \, dA$$

where D is the region in the plane defined by

$$x^4 + y^4 \leq 1, \qquad x \geq 0, \qquad y \geq 0.$$

Hint: Use a change of variables to replace D with a simpler region.

5. Consider the vector field

$$\mathbf{F} = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$$

which is defined on all of the plane except for the origin. Is $\mathbf{F}$ conservative? *Hint:* Try expressing $\mathbf{F} = \nabla f$ with $f(x,y) = g(r)$.

6. Find the tangent plane to the parametrized surface

$$\mathbf{r}(u,v) = \langle v-1, uv, u^3+2 \rangle$$

at the point $(1,2,3)$. Please give an answer of the form $ax+by+cz = d$.

7. Calculate the area of the surface

$$x = yz, \qquad y^2+z^2 \leq 1.$$

8. (a) Sketch the parametrized curve

$$x = \sin\theta\cos\theta, \qquad y = \sin\theta, \qquad 0 \leq \theta \leq 2\pi,$$

including arrows to indicate its orientation.

(b) Calculate the area enclosed by the curve.

9. Calculate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, oriented counterclockwise when viewed from above, and

$$\mathbf{F} = \langle \sin^2 x + y, \sin^2 y + z, xy \rangle.$$

10. Calculate $\iint_S \mathbf{F}\cdot d\mathbf{S}$ where S is the upper hemisphere

$$x^2+y^2+z^2 = 1, \qquad z \geq 0,$$

oriented upward, and

$$\mathbf{F} = \langle xz, yz, z \rangle.$$