Math 53 Final, 12/18/03, 8:00 AM – 11:00 AM

No calculators or notes are permitted. Each of the 12 questions is worth 10 points. Please write your solution to each of the 12 questions on a separate sheet of paper with your name and your TA's name on it. Please put a box around the final answer. To maximize credit, please show your work, and if you have extra time, double check that you got the correct answer and didn't misunderstand the question. Good luck!

1. Let $\mathbf{r}(t)$ be a parametrized curve in the x, y plane satisfying $\mathbf{r}(0) = \langle 1, 2\rangle$ and $\mathbf{r}'(0) = \langle 3, 4\rangle$. Let $f(x, y) = e^{xy}$. Calculate

$$\frac{d}{dt} f(\mathbf{r}(t))\big|_{t=0}.$$

2. (a) Find a normal vector to the surface $x^2 + y^2 + z^2 = 14$ at the point $\langle 3, 2, 1\rangle$.

   (b) Find an equation for the tangent plane to the surface $z = x^2 - y^2$ at the point $\langle 2, 1, 3\rangle$.

3. Find the absolute minimum and maximum values of $f = x^2 - 4x + y^2$ subject to the constraint $x^2 + 2y^2 \leq 1$.

4. Calculate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where S is the portion of the paraboloid

$$z = 9 - x^2 - y^2$$

with $z \geq 0$, oriented using the upward pointing normal, and

$$\mathbf{F} = \langle x, y, z\rangle.$$

5. Calculate

$$\int_{-2}^{2} \int_{y^2}^{4} y \sin(x^2) dx\, dy.$$

6. Find the area of the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 1$ and $z = 2$.

7. Calculate the volume of the region consisting of all points that are inside the sphere $x^2 + y^2 + z^2 = 4$, below the cone $z = \sqrt{x^2 + y^2}$, and above the cone $z = -\sqrt{x^2 + y^2}$.

8. Calculate $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where S is the unit sphere $x^2 + y^2 + z^2 = 1$, oriented using the outward pointing normal, and

$$\mathbf{F} = \langle x + \sin y, y + \sin z, z + \sin x \rangle.$$

9. Either find, or prove that there does not exist:

   (a) a function f on $\mathbb{R}^3$ such that

   $$\nabla f = \langle y, x + z \cos y, \sin y \rangle.$$

   (b) a vector field $\mathbf{F}$ on $\mathbb{R}^3$ such that

   $$\nabla \times \mathbf{F} = \langle z, y, x \rangle.$$

10. Let R be the region $9x^2 + 4y^2 \leq 1$ in the x, y plane. Calculate

$$\int\int_R (9x^2 + 4y^2)^{5/2} dA.$$

11. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the space curve $\mathbf{r}(t) = \langle t, \sin t, \sin t \rangle$, $0 \leq t \leq \pi$, and $\mathbf{F} = \langle x, \sin(\sin y), \cos(\cos z) \rangle$.

12. Let S be the portion of the sphere $x^2 + y^2 + z^2 = 25$ lying above the plane $z = 4$, oriented by the upward pointing normal. Calculate

$$\int\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

for the vector field

$$\mathbf{F} = \langle z^3 - y^3, x^3 - z^3, y^3 - x^3 \rangle.$$