Math 214 HW#7, due 3/17/15 at 12:40 PM

1. Lee 7.16
2. Lee 8.13.
3. Lee 9.6.
4. Lee 9.16.
5. Lee 9.17.

Now let G be a Lie group and let $\mathfrak{g}$ denote its Lie algebra.

6. Recall that the bracket $[\cdot, \cdot] : \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ and the exponential map $\exp : \mathfrak{g} \to G$ are defined using left-invariant vector fields on G. Show that if one uses right-invariant vector fields instead, then the bracket switches sign, while the exponential map is unchanged. *Hint:* Let $f : G \to G$ denote the map sending $g \mapsto g^{-1}$. What does f do to invariant vector fields?

7. (a) Let $X \in \mathfrak{g}$, let X^L denote the left-invariant extension of X to a vector field on G, and let $\{\phi_t\}_{t\in\mathbb{R}}$ denote the flow of X^L. Find and prove a formula showing that the map $\phi_t : G \to G$ agrees with left or right multiplication by an appropriate element of G.

   (b) Suppose that the exponential map $\exp : \mathfrak{g} \to G$ is surjective[1]. Use the result in part (a) to prove that G is abelian if and only if the Lie bracket on $\mathfrak{g}$ vanishes identically.

8. How difficult was this assignment?

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[1]In fact one only needs to assume here that G is connected. This is a weaker condition; see Lee, Exercise 20.6.