Math 214 HW#3, due 2/17/15 at 12:40 PM

1. Let M be a smooth manifold and let $p \in M$. Let I_p denote the set of smooth functions $f : M \to \mathbb{R}$ such that $f(p) = 0$. Let I_p^2 denote the set of sums $\sum_{i=1}^k f_i g_i$ where k is a nonnegative integer and $f_i, g_i \in I_p$. Show that the quotient vector space I_p/I_p^2 is canonically isomorphic to the cotangent space $T_p^* M$.

2. Lee 4.6.

3. Lee 5.1.

4. The *zero section* of the tangent bundle TM is the set of zero tangent vectors,
$$Z = \{(p, 0)\} \subset TM = \{(p, V) \mid p \in M, V \in T_p M\}.$$
   (a) Show that Z is a submanifold of TM which is diffeomorphic to M.
   (b) Show that if $(p, 0) \in Z$, then there is a canonical (not depending on a choice of coordinates) isomorphism
$$T_{(p,0)} TM = T_p M \oplus T_p M.$$

5. The *Hopf fibration* is the map $f : S^3 \to \mathbb{C}P^1$ sending $(z^1, z^2) \in \mathbb{C}^2$ with $|z^1|^2 + |z^2|^2 = 1$ to $[z^1 : z^2] \in \mathbb{C}P^1$. Show that the Hopf fibration is a submersion.

6. Lee 6.15.

7. How difficult was this assignment?