Math 214 HW#1, due 2/3/15 at 12:40 PM

1. Lee 1.7. (Just do the case $n = 2$.)

2. Lee 1.9. (Just do the case $n = 1$.) Also check that the projection $\mathbb{C}^2 \setminus \{0\} \to \mathbb{C}P^1$ is smooth.

3. Show that $\mathbb{C}P^1$ is diffeomorphic to S^2.

4. Consider spherical coordinates on $\mathbb{R}^3$ (not including the line $x = y = 0$) ρ, ϕ, θ defined in terms of the Euclidean coordinates x, y, z by

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi.$$

   (a) Express $\partial/\partial\rho$, $\partial/\partial\phi$, and $\partial/\partial\theta$ as linear combinations of $\partial/\partial x$, $\partial/\partial y$, and $\partial/\partial z$. (The coefficients in these linear combinations will be functions on $\mathbb{R}^3 \setminus (x = y = 0)$.)

   (b) Express $d\rho$, $d\phi$, and $d\theta$ as linear combinations of dx, dy, and dz.

5. Let V and W be finite dimensional vector spaces and let $A : V \to W$ be a linear map. Show that the dual map $A^* : W^* \to V^*$ is given in coordinates as follows. Let $\{e_i\}$ and $\{f_j\}$ be bases for V and W, and let $\{e^i\}$ and $\{f^j\}$ be the corresponding dual bases for V^* and W^*. If $Ae_i = A_i^j f_j$ then $A^* f^j = A_i^j e^i$.

6. Let V be a finite dimensional vector space and let $\langle \cdot, \cdot \rangle$ be an inner product on V. The inner product determines an isomorphism $\phi : V \to V^*$.

   (a) Show that the isomorphism ϕ is given in coordinates as follows. Let $\{e_i\}$ be a basis for V, let $\{e^i\}$ be the dual basis, and write $g_{ij} = \langle e_i, e_j \rangle$. Then $\phi(e_i) = g_{ij} e^j$.

   (b) The inner product, together with the isomorphism ϕ, define an inner product on V^*. Write this in coordinates as $g^{ij} = \langle e^i, e^j \rangle$. Show that the matrix (g^{ij}) is the inverse of the matrix (g_{ij}).

7. Show that if M and N are smooth manifolds and if $p \in M$ and $q \in N$, then there is a canonical isomorphism

$$T_{(p,q)}(M \times N) = T_p M \oplus T_q N.$$

Describe this isomorphism in terms of (a) derivations, (b) linear combinations of partial derivatives with respect to coordinate charts, and (c) velocity vectors of curves.

8. How difficult was this assignment?