Math 214 HW#8, due 4/2/13 at 2:10 PM

1. Grade HW#7.

2. Let E be a smooth vector bundle over a compact smooth manifold M. Show that there exists a positive integer N such that E is isomorphic to a subbundle of the trivial bundle $M \times \mathbb{R}^N$. *Hint:* Modify the first step in the proof of the Whitney embedding theorem.

3. Let M be a smooth manifold, let $f : M \to \mathbb{R}$ be a smooth function, and let $p \in M$. Show that if $V \in T_pM$ satisfies $Vf > 0$, then there exists a Riemannian metric g on M with $\nabla f(p) = V$.

4. Lee 12.1.

5. Lee 13.4. (Just do the case $n = 2$.)

6. Lee 13.10.

7. Lee 13.24.

8. How difficult was this assignment?