Math 185 HW#1, due 2/2/16 at 8:10 AM

Most of this assignment is elementary, although not necessarily easy, and intended to give you practice with complex numbers. Note that if U is an open subset of $\mathbb{C}$ and $f : U \to \mathbb{C}$, we say that f is *holomorphic* if the complex derivative f' is defined at each point in U and continuous. (The book uses the word *analytic* for this. We will see later that if the derivative is defined everywhere then it is automatically continuous.)

1. (a) Calculate $(1-i)^9$.
   (b) Find all square roots of $3+4i$.
   (c) Calculate $(\sqrt{3}+i)^{12}$.

2. Let a and b be distinct complex numbers. If a and b are opposite vertices of a square in the complex plane, what are the other two vertices? Express the answer in terms of addition and multiplication of complex numbers, without referring to the real or imaginary parts of a and b.

3. Let a_1, a_2, a_3 be distinct complex numbers. Show that a_1, a_2, a_3 are the vertices of an equilateral triangle if and only if

$$a_1^2 + a_2^2 + a_3^2 = a_1 a_2 + a_2 a_3 + a_3 a_1.$$

   Hint: Relate both conditions to the condition

$$\frac{a_3 - a_2}{a_2 - a_1} = \frac{a_1 - a_3}{a_3 - a_2}.$$

4. Gamelin, page 5, exercises 6 and 7.

5. Gamelin, page 10, exercise 6.

6. Gamelin, page 14, exercise 2.

7. Prove that $f : \mathbb{C} \to \mathbb{C}$ is holomorphic if and only if the function $g : \mathbb{C} \to \mathbb{C}$ defined by $g(z) = \overline{f(\overline{z})}$ is holomorphic.

8. Let $U \subset \mathbb{C}$ be a connected open set and let $f : U \to \mathbb{C}$ be holomorphic. Show that if $|f|$ is constant, then f is constant.