Math 185 HW#3, due 9/18/12 at 12:40 PM

1. Prove that there is a unique holomorphic function $f : \mathbb{C} \to \mathbb{C}$ such that $f'(z) = f(z)$ and $f(0) = 1$. *Hint:* Let g be another such function and consider the function $h(z) = f(z)g(-z)$. What do you know about $h(z)$?

2. Let log denote the principal branch of the logarithm, which is defined on the complement of the negative real axis and whose values have imaginary part in $(-\pi, \pi)$.

   (a) Show that $\log(zw) - \log(z) - \log(w) \in 2\pi i\mathbb{Z}$.

   (b) Consider the triangle whose vertices are distinct complex numbers a, b, c. Give formulas in terms of log for the angles between the edges of the triangle.

   (c) Use (a) and (b) to show that the sum of the angles in a triangle is π.

3. Gamelin, page 53, exercise 3.

4. Gamelin, page 57, exercises 4, 5, 6, 7.

5. Gamelin, page 62, exercises 2, 4, 5.