Math 113 Homework # 6, due 10/21/9 at 2:10 PM

1. Fraleigh section 14 exercises 34, 37. (Recall that an *automorphism* of G is an isomorphism from G to itself. For each $a \in G$ there is an automorphism i_a defined by $i_a(g) = aga^{-1}$; an automorphism of G is called *inner* if it is i_a for some $a \in G$.)

2. Compute the following quotient groups in terms of the classification of finitely generated abelian groups:

   (a) $\mathbb{Z} \oplus \mathbb{Z}/\langle(6,9)\rangle$

   (b) $\mathbb{Z} \oplus \mathbb{Z}/\langle(4,2),(0,2)\rangle$

3. Fraleigh section 15 exercises 14, 19, 23.

4. Let n be a positive integer and let d be a divisor of n.

   (a) Show that $H = \{R_0, R_d, R_{2d}, \ldots\}$ is a normal subgroup of D_n.

   (b) Describe the cosets of H in D_n.

   (c) Show that $D_n/H \simeq D_d$.

   (d) Show that the commutator subgroup of D_{2n} is $\{R_0, R_2, R_4, \ldots\}$.

   (e) Show that the abelianization D_{2n}^{ab} of D_{2n} is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

5. How challenging did you find this assignment? How long did it take?