Math 113 Homework # 4, due 9/30/9 at 2:10 PM

1. Fraleigh section 8, problem 21; section 9, problem 23.

2. For $f \in S_n$, define the *support* of f to be

$$\mathrm{supp}(f) = \{i \in \{1, \ldots, n\} \mid f(i) \neq i\}.$$

   (a) Show that if $f, g \in S_n$ and $\mathrm{supp}(f) \cap \mathrm{supp}(g) = \emptyset$ then $fg = gf$.

   (b) *Extra credit:* Show that if $f, g \in S_n$ and $|\,\mathrm{supp}(f) \cap \mathrm{supp}(g)| = 1$, then $fgf^{-1}g^{-1}$ is a 3-cycle. (This is used in solving Rubik's cube.)

3. (a) Show that if $\mu = (x_1\ x_2\ \cdots\ x_k) \in S_n$ is a k-cycle and $\sigma \in S_n$ is any permutation then $\sigma\mu\sigma^{-1}$ is the k-cycle

$$\sigma\mu\sigma^{-1} = (\sigma(x_1)\ \sigma(x_2)\ \cdots\ \sigma(x_k)).$$

   (b) Using the above, can you guess a necessary and sufficient condition for two permutations in S_n to be conjugate to each other?

4. Show that every even permutation in A_n can be expressed as a product of (not necessarily disjoint) 3-cycles. *Hint*: use induction on n, imitating one of the proofs given in class that every permutation in S_n is a product of transpositions.

5. Let G be the symmetry group of a cube (with no reflections allowed). Show that $G \simeq S_4$. *Hint:* a cube has four "diagonals" which connect opposite vertices and go through the center of the cube. Any element of G induces a permutation of the set of diagonals. It is enough to show that every permutation of the set of diagonals is realized by some element of G (why?).

6. (a) Find the left and right cosets of the subgroup $H = \{R_0, F_0\}$ of D_4. Are they the same?

   (b) Same question for $H = \{R_0, R_2\}$. (You can use the multiplication table below.)

7. How challenging did you find this assignment? How long did it take?

D_4	R_0	R_1	R_2	R_3	F_0	F_1	F_2	F_3
R_0	R_0	R_1	R_2	R_3	F_0	F_1	F_2	F_3
R_1	R_1	R_2	R_3	R_0	F_1	F_2	F_3	F_0
R_2	R_2	R_3	R_0	R_1	F_2	F_3	F_0	F_1
R_3	R_3	R_0	R_1	R_2	F_3	F_0	F_1	F_2
F_0	F_0	F_3	F_2	F_1	R_0	R_3	R_2	R_1
F_1	F_1	F_0	F_3	F_2	R_1	R_0	R_3	R_2
F_2	F_2	F_1	F_0	F_3	R_2	R_1	R_0	R_3
F_3	F_3	F_2	F_1	F_0	R_3	R_2	R_1	R_0