Math 113 Homework # 3, due 9/23/9 at 2:10 PM

1. Fraleigh section 4, exercise 41.

2. Fraleigh section 5, exercise 13.

3. (a) Fraleigh section 5, exercise 54.

   (b) Is this still true if one replaces intersection by union? Prove or give a counterexample.

4. Let $n > 1$ be an integer and let $\theta = 2\pi/n$. Let P be the regular n-gon with vertices $(\cos i\theta, \sin i\theta)$ for $i \in \mathbb{Z}_n$. The **dihedral group** D_n is the symmetry group of P, which consists of rotations R_i and reflections F_i for $i \in \mathbb{Z}_n$. Here R_i is the counterclockwise rotation around the origin by angle $i\theta$, and F_i is the reflection across the line through the origin and $(\cos i\theta/2, \sin i\theta/2)$.

   Your problem: find (and give at least some justification for) general formulas for R_iR_j, R_iF_j, F_iR_j, and F_iF_j. For example, $R_iR_j = R_{i+j}$, where the addition of indices is mod n.

5. Find all subgroups of D_4.

6. If G is a group, the *center* of G is defined to be

$$Z(G) = \{x \in G \mid xy = yx \text{ for all } y \in G\}.$$

   (a) Show that $Z(G)$ is a subgroup of G.

   (b) For $n > 2$, what is the center of D_n? (Use the multiplication rules you found above. The answer depends on whether n is even or odd.)

7. Fraleigh section 6, exercise 32 (justify as always).

8. How challenging did you find this assignment? How long did it take?