Math 113 Homework # 7, due 3/20/03 at 8:10 AM

1. Fraleigh section 14 exercises 31, 34, 37. (Recall that an *automorphism* of G is an isomorphism from G to itself. For each $a \in G$ there is an automorphism i_a defined by $i_a(g) = aga^{-1}$; an automorphism of G is called *inner* if it is i_a for some $a \in G$.)

2. Fraleigh section 15 exercises 4, 8, 14, 19, 23, 37.

3. Let n be a positive integer and let d be a divisor of n.

   (a) Show that $H = \{R_0, R_d, R_{2d}, \ldots\}$ is a normal subgroup of D_n.

   (b) Describe the cosets of H in D_n.

   (c) Show that $D_n/H \simeq D_d$.

   (d) Show that the commutator subgroup of D_{2n} is $\{R_0, R_2, R_4, \ldots\}$.

   (e) Show that the abelianization D_{2n}^{ab} of D_{2n} is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

4. Extra credit: Let $f, g \in S_n$ be permutations such that $|\operatorname{supp}(f) \cap \operatorname{supp}(g)| = 1$. Show that the commutator $[f, g] = fgf^{-1}g^{-1}$ is a 3-cycle. (This fact is useful for finding 3-cycles to help solve Rubik's cube and related puzzles.)

5. How challenging did you find this assignment? How long did it take?