Math 113 Homework # 4, due 2/20/03 at 8:10 AM

This assignment may be a bit more challenging than the previous one; it's OK if you can't solve every problem. There will be no homework next week, and the first midterm will be on 2/27.

1. Fraleigh section 5, exercise 13 (justify as always) and exercise 54. Also, in $\mathbb{Z}$ we know that $\langle 14\rangle \cap \langle 35\rangle = \langle m\rangle$ for some positive integer m; what is m?

2. If G is a group, the *center* of G is defined to be

$$Z(G) = \{x \in G \mid xy = yx \text{ for all } y \in G\}.$$

   (a) Show that $Z(G)$ is a subgroup of G.

   (b) For $n > 2$, what is the center of D_n? (Use the multiplication rules from the last homework. The answer depends on whether n is even or odd.)

3. Fraleigh section 6, exercise 32 (justify as always) and exercise 44.

4. (a) Let n be a positive integer, let G be a subgroup of $\mathbb{Z}_n$, and let d be the smallest positive integer such that $[d] \in G$. Show that d is a divisor of n.

   (b) Find all subgroups of $\mathbb{Z}_{30}$ and draw the subgroup diagram.

5. For $f \in S_n$, define the *support* of f to be

$$\operatorname{supp}(f) = \{i \in \{1, \ldots, n\} \mid f(i) \neq i\}.$$

   Show that if $f, g \in S_n$ and $\operatorname{supp}(f) \cap \operatorname{supp}(g) = \emptyset$ then $f \circ g = g \circ f$.

6. Let G be the symmetry group of a cube (with no reflections allowed). Show that $G \simeq S_4$. Hint: a cube has four "diagonals" which connect opposite vertices and go through the center of the cube. Any element of G induces a permutation of the set of diagonals.

7. How challenging did you find this assignment? How long did it take?