Math 113 Homework # 3, due 2/13/03 at 8:10 AM

1. Prove the following part of the **Chinese remainder theorem**: if b_1 and b_2 are relatively prime positive integers, and if a_1 and a_2 are any integers, then there exists an integer x such that $x \equiv a_1 \pmod{b_1}$ and $x \equiv a_2 \pmod{b_2}$.

2. Fraleigh section 3, exercises 26, 27, 30, 32.

3. Fraleigh section 4, exercises 19, 20, 28, 41.

4. (a) Let $n > 1$ be an integer and let $\mathbb{Z}_n^*$ be the set of *units* in $\mathbb{Z}_n$, i.e. elements $x \in \mathbb{Z}_n$ such that there exists $y \in \mathbb{Z}_n$ with $xy = 1$. Show that $\mathbb{Z}_n^*$ (with the operation of multiplication) is a group.
   (b) Make multiplication tables for $\mathbb{Z}_8^*$, $\mathbb{Z}_{10}^*$, and $\mathbb{Z}_{12}^*$.
   (c) Show that $\mathbb{Z}_8^* \simeq \mathbb{Z}_{12}^*$ but $\mathbb{Z}_8^* \not\simeq \mathbb{Z}_{10}^*$ and $Z_{10}^* \not\simeq \mathbb{Z}_{12}^*$.

5. (Extra credit) Let $n > 1$ be an integer and let $\theta = 2\pi/n$. Let P be the regular n-gon with vertices $(\cos i\theta, \sin i\theta)$ for $i \in \mathbb{Z}_n$. The **dihedral group** D_n is the symmetry group of P, which consists of rotations R_i and reflections F_i for $i \in \mathbb{Z}_n$. Here R_i is the counterclockwise rotation around the origin by angle $i\theta$, and F_i is the reflection across the line through the origin and $(\cos i\theta/2, \sin i\theta/2)$.

   Your problem: find (and give at least some justification for) general formulas for R_iR_j, R_iF_j, F_iR_j, and F_iF_j. For example, $R_iR_j = R_{i+j}$, where the addition of indices is mod n.

6. How challenging did you find this assignment? How long did it take?