# Math 274: Tropical Geometry

UC Berkeley, Spring 2009
Homework # 4, due Tuesday, February 24

1. Can you recover a matroid $M$ from its tropical linear space $\mathrm{Trop}(M)$? Explain how the flats, bases and circuits of $M$ are encoded in $\mathrm{Trop}(M)$.

2. Let $f$ and $g$ be Laurent polynomials in two variables with generic coefficients, with Newton polygons $P$ and $Q$ respectively. Use tropical geometry to give a proof of Bernstein's Theorem: *The number of solutions of $f = g = 0$ in $(\mathbb{C}^*)^2$ equals* $\mathrm{area}(P) + \mathrm{area}(Q) - \mathrm{area}(P + Q)$. What about the (stable) intersection of $n$ tropical hyperplanes in $\mathbb{R}^n$?

3. Does the *Riemann-Roch Theorem* hold for tropical curves? Find the relevant sources in the literature and summarize what is currently known.

4. Find a "smooth" cubic curve in $\mathbb{TP}^2$ whose *tropical $j$-invariant* equals 17. Can you evaluate the *tropical discriminant* of your cubic polynomial?

5. The following $3 \times 6$-matrix has tropical rank three:
$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 4 & 6 & 8 & 10 \end{pmatrix}.$$
Its tropical column span defines a tropical hexagon in the plane $\mathbb{TP}^2$, while its tropical row span defines a tropical triangle in $\mathbb{TP}^5$. Compute these two objects, draw them, and show that they are isomorphic.

6. In your opinion, is the following statement true or false: *The $4 \times 4$-minors of a $5 \times 5$-matrix are a tropical basis for the ideal they generate.*