Math 274: Tropical Geometry

UC Berkeley, Spring 2009
Homework # 3, due Tuesday, February 10

1. Does there exist a tropical cubic surface in $\mathbb{TP}^3$ that contains 27 distinct tropical lines ? What would be the f-vector of such a cubic surface ?

2. The Grassmannian $G_{2,6}$ is the intersection of the fifteen hypersurfaces $x_{ij}x_{kl} - x_{ik}x_{jl} + x_{il}x_{jk} = 0$ for $1 \leq i < j < k < l \leq 6$. Determine the tropical variety defined by these equations, and compute its f-vector. Show that the vector $w = e_{12} + e_{34} + e_{56}$ lies in this tropical variety, and determine a preimage of w under the valuation map if $K = \mathbb{Q}(t)$.

3. Consider the hypersurface in $\mathbb{P}^4$ given by the parametrization

   $a = u^4 + x^4$, $b = u^3v + x^3y$, $c = u^2v^2 + x^2y^2$, $d = uv^3 + xy^3$, $e = v^4 + y^4$.

   Determine the implicit equation of this hypersurface, compute the corresponding tropical hypersurface, and draw this hypersurface as a graph. Next consider the image in $\mathbb{TP}^4$ of the tropical parametrization

   $$A = U^3 \oplus X^4, B = U^3V \oplus X^3Y, C = U^2V^2 \oplus X^2Y^2, D = UV^3 \oplus XY^3, E = V^4 \oplus Y^4.$$

   Compute this image, and draw it as a subgraph of the previous graph.

4. How do your research interests relate this course, and what do you hope to get out of it ? Please list three (or more) research papers in (or around) tropical geometry that might interest you. Identify one open problem or conjecture in tropical geometry that is interesting to you[1].

[1] Feel free to send me e-mail or drop by my office to discuss these questions.