Math 275: Introduction to Non-Linear Algebra

Bernd Sturmfels, UC Berkeley, Spring 2014
Homework # 5, due Monday, February 24

1. Compute the irreducible polynomial in two variables that vanishes on the 5-*ellipse* with foci $(0,0), (1,0), (2,1), (1,2)$ and $(0,1)$ and radius 7.

2. A calculus student is asked to find the minimum of the polynomial

$$f(x) \;=\; x^6 - 21x^5 + 175x^4 - 735x^3 + 1624x^2 - 1764x + 713.$$

   Express this problem as a semidefinite program (SDP), solve that SDP, and try to draw a picture of the spectrahedron of all feasible solutions.

3. Prove Proposition 7.1 in *CBMS: Solving Systems of Polynomial Eqns.*

4. The discriminant of the characteristic polynomial of a symmetric 3×3-matrix is a homogeneous polynomial Δ of degree six in six variables. How many terms does Δ have? How many faces (of dimensions $0, 1, 2, 3$ $4, 5$ respectively) does the Newton polytope of Δ have? What is the dimension of the real algebraic variety defined by the equation $\Delta = 0$?

5. True or false: Every positive semidefinite symmetric matrix A with entries in the rational numbers has a Cholesky decomposition $A = B \cdot B^T$ where the entries of the real matrix B are expressed in radicals.