Math 275: Introduction to Non-Linear Algebra

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Homework # 4, due Wednesday, February 19

1. *[CBMS 1.8]* Compute all 5 Puiseux series solutions $x(t)$ of the equation
$$x^5 + tx^4 + t^3x^3 + t^6x^2 + t^{10}x + t^{15} \; = \; 0.$$
In each case, guess a formula (in terms of n) for the coefficient of t^n.

2. Let P, Q, R be the three square facets of the 3-cube $[0,1]^3$ adjacent to $(0,0,0)$. Determine the polynomial $V(\lambda) = \text{volume}(\lambda_1 P + \lambda_2 Q + \lambda_3 R)$. Can you generalize your result to d facets of the d-dimensional cube?

3. *[CBMS 3.2]* Draw the Newton polytope of the polynomial
$$f(x_1, x_2, x_3, x_4) \; = \; (x_1 - x_2)(x_1 - x_3)(x_1 - x_4)(x_2 - x_3)(x_2 - x_4)(x_3 - x_4).$$

4. Let P be the square and Q the triangle in our running example. How many distinct mixed subdivisions (as in Figure 3.1) does $P + Q$ have?

5. Find two tetrahedra in $\mathbb{R}^3$ whose Minkowski sum has 16 vertices.

6. *[CBMS 3.4]* Compute the first three terms in each of the four solutions $(x(t), y(t))$ over the Pusieux series $\mathbb{C}\{\{t\}\}$ to the system of two equations
$$\begin{aligned} t^2x^2 + t^5xy + t^{11}y^2 + t^{17}x + t^{23}y + t^{31} \; &= \; 0, \\ t^3x^2 + t^7xy + t^{13}y^2 + t^{19}x + t^{29}y + t^{37} \; &= \; 0. \end{aligned}$$

7. Write the precise statements of Bézout's Theorem and Bernstein's Theorem for n equations in n variables. Derive the former from the latter.