Math 275: Introduction to Non-Linear Algebra

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Homework # 3, due Monday, February 10

1. Give an example of a polynomial ideal I such that I is not radical but I is generated by a set of polynomials all of whose terms are squarefree.

2. Let X be an $n \times n$-matrix of unknowns and consider the ideal $I \subset \mathbb{Q}[X]$ that is generated by the entries of X^n. Determine generators of $\mathrm{Rad}(I)$.

3. Which monomial ideals are primary? Find a combinatorial criterion that is necessary and sufficient. Prove that your criterion is correct.

4. The software `Bertini` can compute primary decompositions numerically. Download this software on your computer and try one example.

5. Which 2×2-matrices X satisfy the equation $X^2 = 2X$?
Formulate this question in the language of primary decomposition and solve it. Same question for 3×3-matrices. How about $n \times n$-matrices?

6. *[CBMS, 5.2]* Let P be a prime ideal and m a positive integer. Show that P is a minimal prime of P^m. Give an example where P^m is not primary.

7. *[CBMS, 5.8]* Find a 3×4 integer matrix with all non-zero entries such that all 3×3-subpermanents are zero. What about larger sizes?

8. *[CBMS 5.11]* For positive integers d and e consider the ideal

$$I \;=\; \langle x_1^d x_2 - x_3^d x_4,\, x_1 x_2^e - x_4^{e+1} \rangle.$$

Find a primary decomposition of I and a minimal generating set for the radical of I. What are the degrees of its generators?